SSC CGL (Tier – II) Mathematics Practice Set

Answers with Explanation

1. (c) $x + \frac{1}{2x} = 2 \Rightarrow 2x + \frac{2}{2x} = 4$

$\Rightarrow 2x + \frac{1}{x} = 4$

On Cubing

$8x^3 + \frac{1}{x^3} + 3.2x.\frac{1}{x}\left(2x + \frac{1}{x}\right) = 64$

$\Rightarrow 8x^3 + \frac{1}{x^3} + 6 \times 4 = 64$

$\Rightarrow 8x^3 + \frac{1}{x^3} = 64 - 24 = 40$

2. (b) sinα + cosβ = 2

sinα á 1 ; cosβ á 1

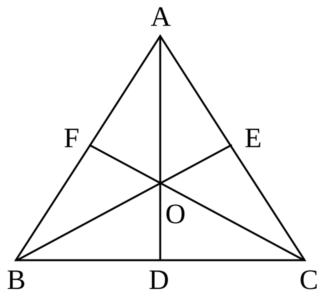
$\Rightarrow$ a = 90° ; β = 0°

$\therefore \sin\left(\frac{2\alpha + \beta}{3}\right) = \sin\left(\frac{180^o}{3}\right)$

$= \sin 60^o = \frac{\sqrt{3}}{2}$

$\cos\frac{\alpha}{3} = \cos 30^o = \frac{\sqrt{3}}{2}$

3. (b)



Area of quadrilateral BDOF = 2 × 15 = 30 sq.cm.

4. (c) $x = 2 - 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

$\Rightarrow x - 2 = 2^{\frac{2}{3}} - 2^{\frac{1}{3}}$

On Cubing

$x^3 - 3x^2 \times 2 + 3x \times 4 - 8$

$= \left(2^{\frac{2}{3}}\right)^3 - \left(2^{\frac{1}{3}}\right)^3 - 3.2^{\frac{2}{3}}.2^{\frac{1}{3}}\left(2^{\frac{2}{3}} - 2^{\frac{1}{3}}\right)$

$\Rightarrow x^3 - 6x^2 + 12x - 8$

$= 4 - 2 - 6(x - 2)$

$\Rightarrow x^3 - 6x^2 + 12x - 8$

$= 2 - 6x + 12$

$\Rightarrow x^3 - 6x^2 + 18x + 18$

$= 2 + 12 + 8 + 18 = 40$

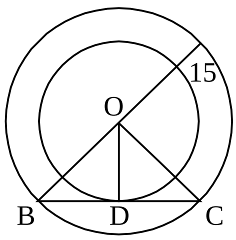
5. (c) cot 10°.cot 80°.cot 20°.cot 70°.cot 60°

= cot10°.tan10.cot 20°. tan 20°. cot 60°

$\left[\because \tan(90^o - \theta) = \cot\theta \quad \tan\theta.\cot\theta = 1\right]$

$= 1.1.\sqrt{3} = \sqrt{3}$

6. (a)



BO = OC = 15 cm.

OD = 9 cm.

$\therefore BD = \sqrt{15^2 - 9^2}$

$= \sqrt{24 \times 6}$ = 12 cm.

$\therefore$ BC = 2 × 12 = 24 cm.

7. (c) $x + \frac{1}{x} = 2$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0 \Rightarrow x = 1$

$\therefore x^{17} + \frac{1}{x^{19}} = 1 + 1 = 2$

8. (a) $\frac{\sin\alpha}{\cos(30^o + \alpha)} = 1$

$\frac{\sin\alpha}{\sin(90^o - 30 - \alpha)} = 1$

$\Rightarrow \frac{\sin\alpha}{\sin(60^o - \alpha)} = 1$

$\Rightarrow \sin\alpha = \sin(60° - \alpha)$

$\Rightarrow \alpha = 60° - \alpha$

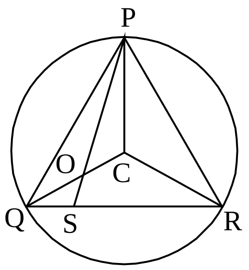
$\Rightarrow 2a = 60° \Rightarrow \alpha = 30°$

$\therefore \sin\alpha + \cos 2\alpha$

$= \sin 30° + \cos 60°$

$= \frac{1}{2} + \frac{1}{2} = 1$

9. (b)



ĎPQS = 60º
ĎQCR = 130º

$\therefore \angle QPR = \frac{1}{2} \times 130^\circ = 65^\circ$

$\Rightarrow$ QPR = 180º – 60º – 65º = 55º

$\therefore$ In aQCR
QC = CR
$\therefore$ ĎCQR = ĎCRQ = 25º
$\therefore$ ĎPQC = ĎQPC = 35º
ĎCPR = 30º
$\therefore$ ĎRPS = 35º

10. (c) $x^2 + y^2 - 4x - 4y + 8 = 0$
$\Rightarrow x^2 - 4x + 4 + y^2 - 4y + 4 = 0$
$\Rightarrow (x-2)^2 + (y-2)^2 = 0$
$\Rightarrow x = 2$ and $y = 2$
$\therefore x - y = 2 - 2 = 0$

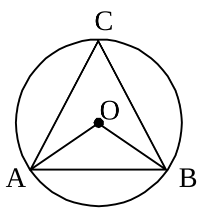
11. (a) $\tan\theta = 1 \Rightarrow \theta = 45^\circ$

$$\therefore \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$$

$$= \frac{8 \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\frac{1}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{7}{\sqrt{2}}}$$

$$= \frac{\frac{13}{\sqrt{2}}}{\frac{13}{2\sqrt{2}}} = \frac{13}{\sqrt{2}} \times \frac{2\sqrt{2}}{13} = 2$$

12. (a)



AO = OB = AB
$\therefore$ ĎAOB = 60°
$\therefore$ ĎACB = 30°

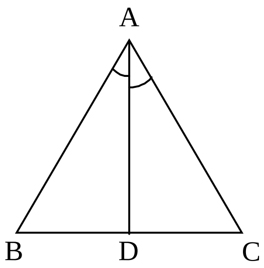
13. (d) $2x + \frac{1}{3x} = 5$
$\Rightarrow 6x^2 + 1 = 15x$
$\Rightarrow 6x^2 + 20x + 1 = 15x + 20x = 35x$

$$\therefore \frac{5x}{6x^2 + 20x + 1} = \frac{5x}{35x} = \frac{1}{7}$$

14. (b) tan4°. tan43°. tan47°. tan86°
= (tan4°. tan86°) (tan43°. 47°)
= (tan4°. cot4°). (tan43°.cot43°) = 1

$$\begin{bmatrix} \tan(90^\circ - \theta) = \cot\theta; \\ \tan\theta . \cot\theta = 1 \end{bmatrix}$$

15. (a)



AD is the internal bisector of $\angle$A.

$$\therefore \frac{AB}{AC} = \frac{BD}{DC} = \frac{5}{7.5 - 2}$$

$$= \frac{5}{2.5} = 2:1$$

16. (b) $x^2 - 3x + 1 = 0$
$\Rightarrow x^2 + 1 = 3x$

$$\Rightarrow x + \frac{1}{x} = 3$$

$$\therefore x^3 + \frac{1}{x^3}$$

$$= \left(x + \frac{1}{x}\right)^3 - 3x.\frac{1}{x}\left(x + \frac{1}{x}\right)$$

$= 27 - 3 \times 3 = 18$

17. (c) $\tan\theta = \frac{4}{3}$ (Given)

$$\therefore \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} = \frac{3\tan\theta + 2}{3\tan\theta - 2}$$

$$= \frac{3 \times \frac{4}{3} + 2}{3 \times \frac{4}{3} - 2} = \frac{4+2}{4-2} = 3$$

18. (c) If the number of sides of the polygon be n, then

$\left(\frac{2n-4}{n}\right)\times 90^o = 144^o$

$\Rightarrow \frac{(2n-4)5}{n} = 8$

$\Rightarrow 10n - 20$

$\Rightarrow 8n$

$\Rightarrow 2n = 20$

$\Rightarrow n = 10$

19. (b) $2x + y = 5$ ...(i)

$x + 2y = 4$ ...(ii)

By equation (i) × 2 – equation (ii), we have

$4x + 2y = 10$

$x + 2y = 4$

– – –

$3x = 6$

$\Rightarrow x = 2$

From equation (i),

$2 \times 2 + y = 5$

$\Rightarrow y = 5 - 4 = 1$

$\therefore$ Point of intersection = (2, 1)

20. (d) $\sec 17° - \sin 73°$

$= \sec 17° - \sin(90° - 17°)$

$= \sec 17° - \cos 17°$

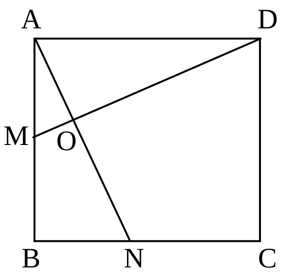
$= \frac{1}{\cos 17^o} - \cos 17^o$

$= \frac{1-\cos^2 17^o}{\cos 17^o} = \frac{\sin^2 17}{\cos 17^o}$

$= \frac{\frac{x^2}{y^2}}{\sqrt{1-\frac{x^2}{y^2}}}$

$= \frac{\frac{x^2}{y^2}}{\sqrt{\frac{y^2-x^2}{y}}} = \frac{x^2}{y\sqrt{y^2-x^2}}$

21. (b)



If AB = 2x, then BN = x

$\therefore AN = \sqrt{4x^2+x^2} = \sqrt{5}x$

Similarly,

$MD = \sqrt{4x^2+x^2} = \sqrt{5}x$

22. (c) $x - \frac{1}{x} = 5$

On squaring both sides,

$x^2 + \frac{1}{x^2} - 2 = 25$

$\Rightarrow x^2 + \frac{1}{x^2} = 27$

23. (a) tanl° . tan2° . tan3°... tan45°... tan88° tan89°

= (tanl° . tan89°) (tan2° . tan88°) ... tan45°

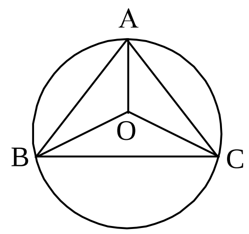
= (tanl° . cotl°).(tan2° . cot2°) ... tan45° = 1

[$\because \tan(90^o - \theta)$

$= \cot\theta . \tan\theta . \cot\theta = 1$]

24. (c) $\because \angle BAC = 85^o$

$\therefore \angle BOC = 2 \times 85^o = 170^o$



$\therefore \angle OBC = \angle OCB = 5^o$

$\therefore \angle OCA = \angle OAC = 75^o - 5^o = 70^o$

25. (b) $x = 3 + 2\sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{3+2\sqrt{2}}$

$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$

$\therefore \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$

$= 3 + 2\sqrt{2} + 3 - 2\sqrt{2} - 2 = 4$

$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = 2$

26. (c) $A + B + C = \pi$

$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$$\Rightarrow \sin\left(\frac{A+B}{2}\right)$$

$$\Rightarrow \sin\left(\frac{\pi}{2}-\frac{C}{2}\right)=\cos\frac{C}{2}$$

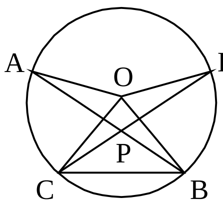
Similarly,

$$\cos\left(\frac{A+B}{2}\right)=\sin\frac{C}{2}$$

$$\cot\left(\frac{A+B}{2}\right)=\tan\frac{C}{2}$$

$$\tan\left(\frac{A+B}{2}\right)=\cot\frac{C}{2}$$

27. (c)



Join CB.
ĎAOC + ĎBOD
= 2ĎABC + 2ĎBCD
(Exterior angles of triangle)
= 2 (ĎABC + ĎBCD)
= 2ĎBPD

$$\therefore \angle BPD=\frac{1}{2}\ (50^o + 40^o) = 45^o$$

28. (a) $\frac{a}{3}=\frac{b}{2}$

$$\Rightarrow \frac{a}{b}=\frac{3}{2}$$

$$\therefore \frac{2a+3b}{3a-2b}=\frac{2\times\frac{a}{b}+3}{3\times\frac{a}{b}-2}$$

$$=\frac{2\times\frac{3}{2}+3}{3\times\frac{3}{2}-2}=\frac{6}{\frac{9-4}{2}}=\frac{12}{5}$$

29. (c) $(\sec A-\cos A)^2+(\operatorname{cosec}A-\sin A)^2-(\cot A-\tan A)^2$

$=\sec^2\mathbf{A}+\cos^2A-2\sec A\cos A+\operatorname{cosec}^2A+\sin^2A-2\operatorname{cosec}A.\sin A-\cot^2A-\tan^2A+2\cot A.\tan A$

$=\sec^2A-\tan^2A+\cos^2A+\sin^2A+\operatorname{cosec}^2A-\cot^2A-2$

$= 3-1 = 1.$

$$\left[\begin{array}{l}\because \sec A.\cos A=1;\\ \sin A.\operatorname{cosec}A=1;\\ \tan A.\cot A=1\text{etc.}\end{array}\right]$$

30. (b) The largest chord of a circle is its diameter.

31. (b) $x^2 + x + 1$

$$= x^2 + 2.x.\frac{1}{2}+\frac{1}{4}+\frac{3}{4}$$

$$=\left(x+\frac{1}{2}\right)^2+\left(\pm\frac{\sqrt{3}}{2}\right)^2$$

$$\therefore \left(x+\frac{1}{2}\right)^2+\left(\pm\frac{\sqrt{3}}{2}\right)^2$$

$$=\left(x+\frac{1}{2}\right)^2+q^2$$

$$\Rightarrow q=\pm\frac{\sqrt{3}}{2}$$

32. (b) $a^2 - 4a - 1 = 0$

$\Rightarrow a^2 - 1 = 4a$

On dividing by a, we have

$$a-\frac{1}{a}=4$$

$$\therefore a^2+\frac{1}{a^2}+3\left(a-\frac{1}{a}\right)$$

$$=\left(a-\frac{1}{a}\right)^2+2+3\left(a-\frac{1}{a}\right)$$

$= 16 + 2 + 3\ (4) = 30$

33. (c) $x=\sqrt[3]{a+\sqrt{a^2+b^3}}+\sqrt[3]{a-\sqrt{a^2+b^3}}$

Cubing both sides,

$$x^3=\left(\sqrt[3]{a+\sqrt{a^2+b^3}}\right)^3+\left(\sqrt[3]{a-\sqrt{a^2+b^3}}\right)^3+$$

$$3\left(\sqrt[3]{a+\sqrt{a^2+b^3}}\right)\left(\sqrt[3]{a-\sqrt{a^2+b^3}}\right)$$

$$\left(\begin{array}{l}\sqrt[3]{a+\sqrt{a^2+b^3}}\\ +\sqrt[3]{a-\sqrt{a^2+b^3}}\end{array}\right)$$

$$=a+\sqrt{a^2+b^3}+a-\sqrt{a^2+b^3}$$

$$+3\left(\begin{array}{l}a+\sqrt{a^2+b^3}\times\\ a-\sqrt{a^2+b^3}\end{array}\right)^{\frac{1}{3}}x$$

$= 2a + 3\left(a^2 - a^2 - b^3\right)^{\frac{1}{3}} x$

$= 2a + (-3bx)$

$\therefore\ x^3 + 3bx = 2a$

34. (c) $\dfrac{1}{1+2^{a-b}} + \dfrac{1}{1+2^{b-a}}$

$= \dfrac{1}{1+\dfrac{2^a}{2^b}} + \dfrac{1}{1+\dfrac{2^b}{2^a}}$

$= \dfrac{2^b}{2^b+2^a} + \dfrac{2^a}{2^a+2^b} = \dfrac{2^b+2^a}{2^b+2^a} = 1$

35. (a) $x + \dfrac{1}{4x} = \dfrac{3}{2}$

$\Rightarrow\ 2x + \dfrac{1}{2x} = 3$

Cubing both sides,

$8x^3 + \dfrac{1}{8x^3} + 3 \times 2x \times \dfrac{1}{2x}$

$\left(2x + \dfrac{1}{2x}\right) = 27$

$\Rightarrow 8x^3 + \dfrac{1}{8x^3} + 3 \times 3 = 27$

$\Rightarrow 8x^3 + \dfrac{1}{8x^3} = 27 - 9 = 18$

36. (a) No. of terms in 1 + 5 + 9 + ... + 89 = n

$\therefore\ a + (n-1)d = t_n$

$\Rightarrow\ 1 + (n-1)4 = 89$

$\Rightarrow\ (n-1)4 = 89 - 1 = 88$

$\Rightarrow\ n - 1 = 22$

$\Rightarrow\ n = 23$

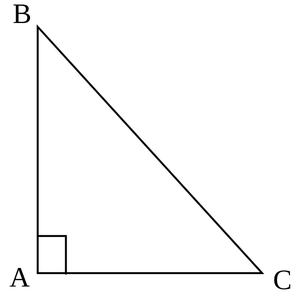
Now, $\sin^2 1^o + \sin^2 89^o + \sin^2 5^o + \sin^2 85^o + \ldots$ + to 22 terms + $\sin^2 45^o$

$= (\sin^2 1^o + \cos^2 1^o) + (\sin^2 5^o + \cos^2 5^o) + \ldots +$ to 11 terms + $\left(\dfrac{1}{\sqrt{2}}\right)^2$

$= 11 + \dfrac{1}{2} = 11\dfrac{1}{2}$

$\left[\begin{array}{l}\sin(90^o - \theta) = \cos\theta \\ \sin^2\theta + \cos^2\theta = 1\end{array}\right]$

37. (b)



If AB = x, BC = 2x units

$\therefore$ AC $= \sqrt{4x^2 - x^2} = \sqrt{3}x$

$\therefore \sin ACB = \dfrac{AB}{BC} = \dfrac{1}{2} = \sin 30^o$

$\therefore \angle ACB = 30^o$

38. (d) $(a-1)^2 + (b+2)^2 + (c+1)^2 = 0$

$\Rightarrow\ a - 1 = 0 \Rightarrow a = 1;$

$b + 2 = 0 \Rightarrow b = -2$

$c + 1 = 0 \Rightarrow c = -1$

$\therefore\ 2a - 3b + 7c$

$= 2 - 3(-2) + 7(-1)$

$= 2 + 6 - 7 = 1$

39. (b) $2\sin^2\theta + 3\cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$

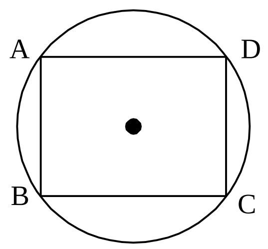
$= 2(2\sin^2\theta + \cos^2\theta) + \cos^2\theta$

$= 2 + \cos^2\theta$

$\because$ Minimum value of $\cos\theta = -1$

$\therefore$ Required minimum value = 2 + 1 = 3

40. (d)



ABCD is a cyclic parallelogram.

$\therefore \angle B + \angle D = 180^o$

$\Rightarrow\ 2\angle B = 180^o$

$\Rightarrow \angle B = 90^o$

41. (c) If a + b + c = 0,

then, $a^3 + b^3 + c^3 = 3abc$

Here, $y - z + z - x + x - y = 0$

$\therefore\ (y-z)^3 + (z-x)^3 + (x-y)^3$

$= 3(y-z)(z-x)(x-y)$

42. (c) $\dfrac{1}{\operatorname{cosec}^2 51^o} + \sin^2 39^o + \tan^2 51^o - \dfrac{1}{\sin^2 51^o . \sec^2 39^o}$

$= \sin^2 51^o + \sin^2 39^o + \tan^2(90^o - 39^o) -$

$\frac{1}{\sin^2(90^o - 39^o).\sec^2 39^o}$

$= \cos^2 39^o + \sin^2 39^o + \cot^2 39^o - \theta$

$[\because \sin(90^o - \theta) = \cos\theta]$

$\tan(90^o - \theta) = \cot\theta]$

$= 1 + \cot^2 39^o - 1$

$= \operatorname{cosec}^2 39^o - 1 = x^2 - 1$

43. (b) If the number of sides of regular polygon be n, then

$\frac{(2n-4)90^o}{n} = \frac{360}{n} \times 3$

$\Rightarrow 2n - 4 = 4 \times 3$

$\Rightarrow 2n = 12 + 4 = 16$

$\therefore n = 8$

44. (a) $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} - 4 = 0$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 + y^2 + \frac{1}{y^2} - 2 = 0$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 + \left(y - \frac{1}{y}\right)^2 = 0$

$\Rightarrow x - \frac{1}{x} = 0$

$\Rightarrow x^2 - 1 = 0 \Rightarrow x = 1$

Similarly,

$y = 1$

$\therefore x^2 + y^2 = 1 + 1 = 2$

45. (b) $\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = \frac{2}{1}$

By componendo and dividendo

$\frac{2\tan\theta}{2\cot\theta} = \frac{3}{1}$

$\Rightarrow \frac{\sin\theta}{\cos\theta}.\frac{\sin\theta}{\cos\theta} = 3$

$\Rightarrow \sin^2\theta = 3\cos^2\theta$

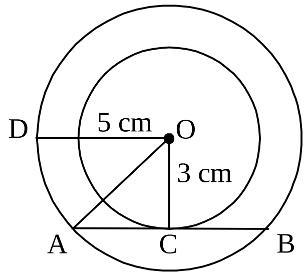
$\Rightarrow \sin^2\theta = 3(1 - \sin^2\theta)$

$\Rightarrow 4\sin^2\theta = 3$

$\Rightarrow \sin^2\theta = \frac{3}{4}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

46. (c)



$AC = \sqrt{AO^2 - OC^2}$

$= \sqrt{5^2 - 3^2}$

$= \sqrt{25 - 9} = \sqrt{16} = 4\,cm$

$\therefore AB = 2 \times 4 = 8$ cm

47. (a) $x + \frac{1}{x} = \sqrt{3}$

Cubing both sides,

$x^3 + \frac{1}{x^3} = 3\left(x + \frac{1}{x}\right) = (\sqrt{3})^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} = 0$

Now, $x^{18} + x^{12} + x^6 + 1$

$= x^{12}(x^6 + 1) + 1(x^6 + 1)$

$= (x^{12} + 1)(x^6 + 1)$

$= (x^{12} + 1).x^3\left(x^3 + \frac{1}{x^3}\right) = 0$

48. (d) $\sin\theta = \cos(90° - \theta)$;

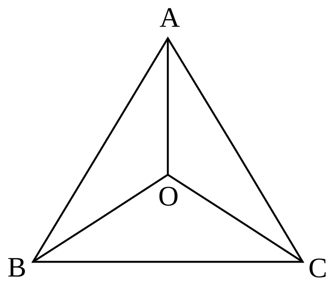
$\sin(90° - \theta) = \cos\theta$

$\therefore \sin 85° = \sin(90° - 5°) = \cos 5°$

$\therefore (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°)$ + ... to 8 terms + $\sin^2 45° + \sin^2 90°$

$= 8 \times 1 + \frac{1}{2} + 1 = 9\frac{1}{2}$

49. (b)



ĎBOC $= 90^o + \frac{1}{2}$ ĎBAC

$= 90^o + 15^o = 105^o$

50. (d) $(ad - bc)^2 + (ac + bd)^2$
$= a^2d^2 + b^2c^2 - 2abcd + a^2c^2 + b^2d^2 - 2abcd$
$= a^2d^2 + b^2c^2 + a^2c^2 + b^2d^2$
$= a^2d^2 + b^2d^2 + b^2c^2 + a^2c^2$
$= d^2 (a^2 + b^2) + c^2 (b^2 + a^2)$
$= (a^2 + b^2) (c^2 + d^2)$
$= 2 \times 1 = 2$

51. (c) $2 \cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$

$2\sin\theta + \cos\theta = x$ (Let)
On squaring and adding,
$4\cos^2\theta + \sin^2\theta - 4\sin\theta.\cos\theta + 4\sin^2\theta + \cos^2\theta + 4\sin\theta.\cos\theta$

$= \frac{1}{2} + x^2$

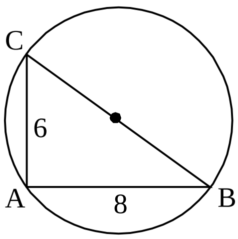
$\Rightarrow 4(\cos^2\theta + \sin^2\theta) + (\cos^2\theta + \sin^2\theta)$

$= \frac{1}{2} + x^2$

$\Rightarrow \frac{1}{2} + x^2 = 5$

$\Rightarrow x^2 = 5 - \frac{1}{2} = \frac{9}{2} \Rightarrow x = \frac{3}{\sqrt{2}}$

52. (d)



ĎBAC = 90º
$\therefore$ BC is the diameter of the circle.

$\therefore BC = \sqrt{AB^2 + AC^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$

$= \sqrt{100} = 10$ cm

$\therefore$ Radius of the circle = 5 cm

53. (a) $\frac{5x-3}{x} + \frac{5x-3}{y} + \frac{5z-3}{z} = 0$

$\Rightarrow \frac{5x}{x} - \frac{3}{x} + \frac{5y}{y} - \frac{3}{y} + \frac{5z}{z} - \frac{3}{z} = 0$

$\Rightarrow \frac{3}{x} + \frac{3}{y} + \frac{3}{z} = 15$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{15}{3} = 5$

54. (c) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$

$\Rightarrow \sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$
$\Rightarrow 4\cos\theta = 2\sin\theta \Rightarrow \tan\theta = 2$
$\therefore \sin^4\theta - \cos^4\theta$
$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$
$= \sin^2\theta - \cos^2\theta$
$= \cos^2\theta (\tan^2\theta - 1)$

$= \frac{\tan^2\theta - 1}{1 + \tan^2\theta} = \frac{4-1}{1+4} = \frac{3}{5}$

55. (b) Length of the rubber band = $3d + 2\pi r$
$= (30 + 10\pi)$ cm

56. (d) $x^4 + \frac{1}{x^4} = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 121$

$\Rightarrow x^2 + \frac{1}{x^2} = 11$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = 11$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 9 \Rightarrow x - \frac{1}{x} = 3$

Cubing both sides,

$\left(x - \frac{1}{x}\right)^3 = 27$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 27$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 3 = 27$

$\Rightarrow x^3 - \frac{1}{x^3} = 27 + 9 = 36$

57. (d) $\frac{\sin 39^\circ}{\cos 51^\circ}$ + 2 tan 11°.tan79°. tan31°.tan59° .tan45° – 3(sin²21° + sin² 69°)

$= \frac{\sin 39^\circ}{\cos(90^\circ - 39^\circ)}$ + 2 tan l1°. tan (90° – 11°). tan3l°. tan (90° – 59°) . 1 – 3 (sin² 21° + sin² (90° – 21°)

$= \frac{\sin 39^o}{\sin 39^o} + 2 \tan 11°. \cot 11°. \tan 31°. \cot 31^0 -$ $3(\sin^2 21° + \cos^2 21^o) = 1 + 2 - 3 = 0$

[∵ ... $\theta.\cot\theta = 1, \sin^2\theta + \cos^2\theta = 1$]

58. (c) The chord nearer to the centre is larger.

$\therefore \frac{15}{8} = \frac{x}{16}$

$\Rightarrow x = \frac{15 \times 16}{8} = 30$ cm

59. (d) $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy}$

$= \frac{x^3 + y^3 + z^3}{xyz} = \frac{3xyz}{xyz} = 3$

60. (c) $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$

$\Rightarrow \cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$

$\Rightarrow 4\cos^2\theta = 3\cot^2\theta = 3\frac{\cos^2\theta}{\sin^2\theta}$

$\Rightarrow 4\cos^2\theta - 3\frac{\cos^2\theta}{\sin^2\theta} = 0$

$\Rightarrow \cos^2\theta\left(4 - \frac{3}{\sin^2\theta}\right) = 0$

$\therefore 4 - \frac{3}{\sin^2\theta} = 0$

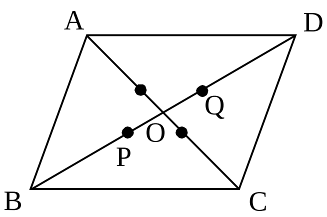
$\Rightarrow 4\sin^2\theta = 3$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2} = \sin 60^o$

$\Rightarrow \theta = 60^o$

$\cos^2\theta = 0 \Rightarrow \theta = 90^o$

61. (b)



Centroid is the point where medians intersect. Diagonals of parallelogram bisect each other.

$OP = \frac{1}{3} \times 9 = 3$ cm

$OQ = \frac{1}{3} \times 9 = 3$ cm

$\therefore$ PQ = 6 cm

62. (a) $2^x = 3^y = 6^{-z} = k$

$\Rightarrow 2 = k^{\frac{1}{x}}; 3 = k^{\frac{1}{y}}; 6 = k^{-\frac{1}{z}}$

$\because 2 \times 3 = 6$

$\Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}}$

$\Rightarrow k^{\frac{1}{x}+\frac{1}{y}} = k^{-\frac{1}{z}}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} = -\frac{1}{z} \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

63. (c) A = tan 11° . tan 29°

B = 2 cot 61° . cot 79°

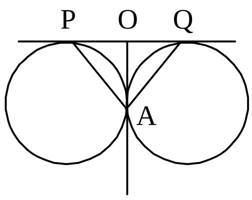
= 2 cot (90° – 29°) cot (90° – 11°)

= 2 tan 29° . tan 11°

[$\because$ cot (90° – θ) = tan θ]

= 2A

64. (b)



AO is perpendicular to PQ.

OA = OP = OQ.

ĎOPA = ĎOAP

= ĎOQA = 45º

$\therefore \angle PAQ = 45^o + 45^o = 90^o$

65. (c) $\frac{x}{a} = b - c; \frac{y}{b} = c - a; \frac{z}{c} = a - b$

Again, b – c + c – a + a – b = 0

$\therefore \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$

$= (b - c)^3 + (c - a)^3 + (a - b)^3$

$= 3(b - c)(c - a)(a - b)$

$= \frac{3xyz}{abc}$

66. (d) $(\sec x . \sec y + \tan x . \tan y)^2 - (\sec x . \tan y + \tan x . \sec y)^2$

$= \sec^2 x. \sec^2 y + \tan^2 x . \tan^2 y + 2 \sec x . \sec y . \tan x . \tan y - \sec^2 x . \tan^2 y - \tan^2 x . \sec^2 y - 2 \sec x . \sec y . \tan x . \tan y$

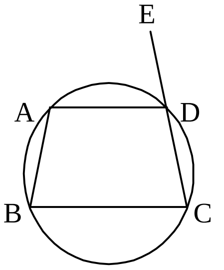
$= \sec^2 x . \sec^2 y + \tan^2 x . \tan^2 y - \sec^2 x . \tan^2 y - \tan^2 x . \sec^2 y$

$= \sec^2 x . \sec^2 y - \sec^2 x . \tan^2 y - \tan^2 x . \sec^2 y + \tan^2 x . \tan^2 y$

$= \sec^2 x (\sec^2 y - \tan^2 y) - \tan^2 x (\sec^2 y - \tan^2 y)$

$= \sec^2 x - \tan^2 x = 1$

67. (d)



$\angle ABC + \angle CDA = 180°$
$\Rightarrow \angle CDA = 180° - 72° = 108°$
AD || BC
$\angle BCD = \angle ADE = \angle ABC = 72°$

68. (d) $x^2 \geq 0$

$\therefore$ Minimum value $= 0 + \frac{1}{1} - 3 = -2$

69. (b) $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

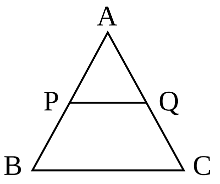
$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 1 \Rightarrow \operatorname{cosec}\theta = 1$

$\therefore \sin^{100}\theta + \operatorname{cosec}^{100}\theta$
$= 1 + 1 = 2$

70. (b)



$\frac{AP}{PB} = \frac{AQ}{QC} = \frac{1}{2}$

$\Rightarrow \frac{QC}{AQ} = \frac{2}{1} \Rightarrow \frac{QC+AQ}{AQ} = \frac{3}{1}$

$\Rightarrow AC = 3AQ = 9$ cm

71. (b) $3x + \frac{1}{2x} = 5$

On multiplying both sides by $\frac{2}{3}$,

$2x + \frac{1}{3x} = \frac{10}{3}$

Cubing both sides,

$8x^3 + \frac{1}{27x^3} + 3 \times 2x \times \frac{1}{3x}\left(2x + \frac{1}{3x}\right) = \frac{1000}{27}$

$\Rightarrow 8x^3 + \frac{1}{27x^3} + 2 \times \frac{10}{3} = \frac{1000}{27}$

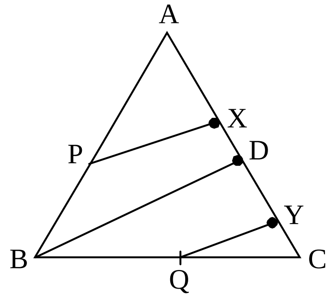
$\Rightarrow 8x^3 + \frac{1}{27x^3} = \frac{1000}{27} - \frac{20}{3}$

$= \frac{1000-180}{27} = \frac{820}{27} = 30\frac{10}{27}$

72. (b) $\tan 7° \,.\, \tan 23° \,.\, \tan 60° \,.\, \tan 67° \,.\, \tan 83°$
$= \tan 7° \,.\, \tan 83° \,.\, \tan 23° \,.\, \tan 67° \,.\, \tan 60°$
$= \tan 7° \,.\, \tan(90° - 7°) \,.\, \tan 23° \,.\, \tan(90° - 23°) \,.\, \tan 60°$
$= \tan 7° \,.\, \cot 7° \,.\, \tan 23° \,.\, \cot 23° \,.\, \tan 60°$
$= 1.1.\sqrt{3} = \sqrt{3}$

$\left[\begin{array}{l} \tan 90° - \theta = \cot\theta \\ \tan\theta.\cot\theta = 1 \end{array}\right]$

73. (b)



PX || BD and PX $= \frac{1}{2}$ BD

QY || BD and QY $= \frac{1}{2}$ BD

$\therefore$ PX : QY = 1 : 1

74. (c) $(a-3)^2 + (b-4)^2 + (c-9)^2 = 0$
$\Rightarrow a - 3 = 0 \Rightarrow a = 3$
$b - 4 = 0 \Rightarrow b = 4$
and $c - 9 = 0$
$\Rightarrow c = 9$

$\therefore \sqrt{a+b+c}$
$= \sqrt{3+4+9}$
$= \sqrt{16} = \pm 4$

75. (a) $\tan\theta + \cot\theta = 2$

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

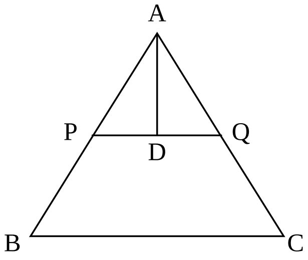
$\Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta = 1$

$\therefore \cot\theta = \frac{1}{\tan\theta} = 1$

$\therefore \tan^{100}\theta + \cot^{100}\theta = 1 + 1 = 2$

76. (c)



PQ || BC

ĎAPQ = ĎABC = 60º

ĎAQP = ĎACB = 60º

$\therefore$ Area of aAPQ

$= \frac{\sqrt{3}}{4} \times (PQ)^2$

$= \frac{\sqrt{3}}{4} \times (5)^2 = \frac{25\sqrt{3}}{4}$ sq.cm.

77. (a) $2^{x+3} = 32 = 2^5$

$\Rightarrow x + 3 = 5$

$\Rightarrow x = 5 - 3 = 2$

$\therefore 3^{X+1} = 3^3 = 27$

78. (b) $\sec\theta = \frac{4x^2+1}{4x}$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

$= \sqrt{\left(\frac{4x^2+1}{4x}\right)^2 - 1}$

$= \sqrt{\frac{(4x^2+1)^2 - (4x)^2}{(4x)^2}}$

$= \frac{(2x+1)(2x-1)}{4x} = \frac{4x^2-1}{4x}$

$\therefore \sec\theta + \tan\theta = \frac{4x^2+1}{4x} + \frac{4x^2-1}{4x}$

$= \frac{4x^2+1+4x^2-1}{4x}$

$= \frac{8x^2}{4x} = 2x$

79. (b) Each interior angle

$= \left(\frac{2n-4}{n}\right) \times 90^o$

$\therefore \frac{(2n-4) \times 90^o}{n} = 105^o$

$\Rightarrow (12n - 4) \times 6 = 7n$

$\Rightarrow 12n - 24 = 7n$

$\Rightarrow 5n = 24$

$n = \frac{24}{5}$ which is impossible.

80. (b) $x + \frac{1}{x} = 2$

$\Rightarrow x^2 + 1 = 2x$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0$

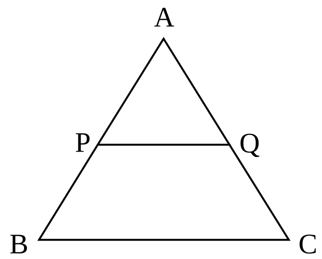
$\Rightarrow x = 1$

$\therefore x^2 + \frac{1}{x^3} = 1 + 1 = 2$

81. (c) Sum of remaining two angles $= \pi - \frac{5\pi}{9} = \frac{4\pi}{9}$

$\therefore$ Each angle $= \frac{1}{2} \times \frac{4\pi}{9} = \frac{2\pi}{9}$

82. (d)



aAPQ ~ aABC

$\therefore \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$

Now, $\frac{AP}{PB} = \frac{3}{1}$

$\Rightarrow \frac{AB}{AB - AP} = \frac{3}{1}$

$\Rightarrow \frac{AB - AP}{AB} = \frac{1}{3}$

$\Rightarrow 1 - \frac{AP}{AB} = \frac{1}{3}$

$\Rightarrow \frac{AP}{AB} = 1 - \frac{1}{3} = \frac{2}{3} = \frac{PQ}{BC}$

83. (a) $\frac{a}{b} + \frac{b}{a} = 1$

$\Rightarrow \frac{a^2 + b^2}{ab} = 1$

$\Rightarrow a^2 + b^2 = ab$

$\Rightarrow a^2 + b^2 - ab = 0$

$\therefore a^3 + b^3$

$= (a + b)(a^2 - ab + b^2) = 0$

84. (c) $\cos x + \cos^2 x = 1$

$\Rightarrow \cos x = 1 - \cos^2 x = \sin^2 x$

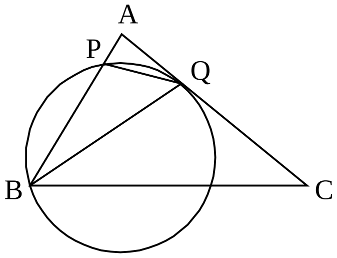
. $\therefore \sin^{12}x + 3\sin^{10}x + 3\sin^8x + \sin^6x - 1$

$= (\sin^4 x + \sin^2 x)^3 - 1$

$= (\cos^2 x + \sin^2 x)^3 - 1$

$= 1 - 1 = 0$

85. (d)



AB = AC = 2x
AQ = QC = x
AB is a secant.

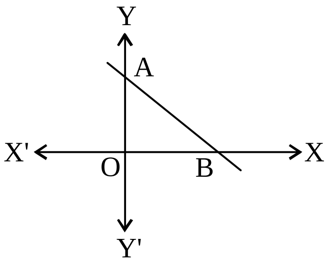
$\Rightarrow$ AP × AB = AQ^2

$\Rightarrow$ AP × 2x = x^2

$\Rightarrow AP = \frac{x}{2}$

$\therefore \frac{AP}{AB} = \frac{x}{2 \times 2x} = \frac{1}{4}$

86 (c)

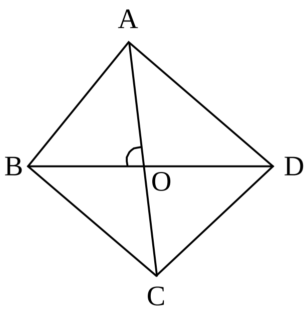


Putting y = 0 in the equation 3x + 4y = 12,
3x + 0 = 12 $\Rightarrow$ x = 4
Co-ordinates of point B = (4, 0)
Putting x = 0 in the equation 3x + 4y = 12
0 + 4y = 12
0 + 4y = 12 $\Rightarrow$ y = 3
$\therefore$ Co-ordinates of point A = (0, 3)
$\therefore$ OB = 4 and OA = 3

$\therefore$ Area of aOAB $= \frac{1}{2} \times OB \times OA$

$= \frac{1}{2} \times 4 \times 3 = 6$ sq. units

87. (a)



$\angle BAD = 60^o$

$\therefore \angle BAO = 30^o$

$\angle ABO = 60^o$

$\therefore \sin 60^o = \frac{OA}{AB}$

$\Rightarrow \frac{\sqrt{3}}{2} \times 8 = OA$

$\therefore OA = 4\sqrt{3}$

$\therefore AC = 8\sqrt{3}$ metre

88. (c) $0 = \frac{s}{r}$

$\Rightarrow$ s = r0

$\Rightarrow$ s = $r_1 0_1 = r_2 0_2$

$\Rightarrow \frac{r_1}{r_2} = \frac{0_2}{0_2} = \frac{75}{60} = \frac{5}{4}$

89. (d) If x = y = z, then

$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{3}{x^2}$ and $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$

$= \frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2} = \frac{3}{x^2}$

90. (b) $\sin\theta$ + $\mathrm{cosec}\,\theta$ = 2

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

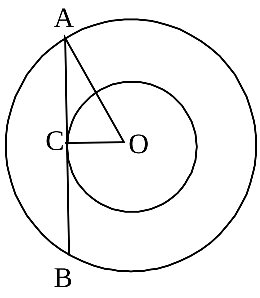
$\Rightarrow (\sin\theta - 1)^2 = 0$

$\Rightarrow \sin\theta = 1$

$\therefore \mathrm{cosec}\,\theta = 1$

$\therefore \sin^9\theta + \mathrm{cosec}^9\theta = 1 + 1 = 2$

91. (c)



$OC = \sqrt{3} - 1$

$OA = \sqrt{3} + 1$

$AC = \sqrt{\left(\sqrt{3}+1\right)^2 - \left(\sqrt{3}-1\right)^2}$

$= \sqrt{4\sqrt{3}} = 2\sqrt[4]{3}$

$\therefore AB = 4.\sqrt[4]{3}$ cm

92. (c) $a + b + c = 0$

$\Rightarrow a + b = -c;\ b + c = -a,\ c + a = -b$

$\therefore \frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}$

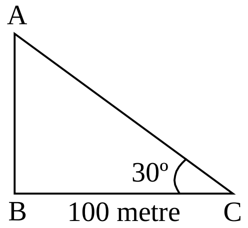
$= -1 - 1 - 1 = -3$

$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$

$= -1 - 1 - 1 = -3$

$\therefore$ Expression $= -3 \times -3 = 9$

93. (a)



AB = Tower = h metre

$\angle ACB = 30^o$;

BC = 100 metre

$\therefore \tan 30^o = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100}$

$\Rightarrow h = \frac{100}{\sqrt{3}}$ metre

94. (c) Sum of interior angles

$= (2n - 4) \times 90°$

$= (2 \times 5 - 4) \times 90° = 540°$

$\therefore 2x + 3x + 3x + 5x + 5x = 540°$

$\Rightarrow 18x = 540° \Rightarrow x = 30°$

$\therefore$ Smallest angle $= 2x° = 60°$

95. (a) $a + \frac{1}{b} = 1 \Rightarrow ab + 1 = b$

$\Rightarrow ab = b - 1$ .....(i)

Again,

$b + \frac{1}{c} = 1$

$\frac{1}{c} = 1 - b$ .....(ii)

On multiplying

$abc = \frac{b-1}{1-b} = -1$

96. (a) Third proportional of a and b $= \frac{b^2}{a}$

$= \frac{\left(\sqrt{x^2 + y^2}\right)^2}{\frac{x}{y} + \frac{y}{x}} = \frac{x^2 + y^2}{\frac{x^2 + y^2}{xy}} = xy$

97. (c) $\left(\frac{3}{4}\right)^3 \times \left(\frac{4}{3}\right)^{-7} = \left(\frac{3}{4}\right)^{2x}$

$\Rightarrow \left(\frac{3}{4}\right)^3 \times \left(\frac{3}{4}\right)^7 = \left(\frac{3}{4}\right)^{2x}$

$\Rightarrow \left(\frac{3}{4}\right)^{10} = \left(\frac{3}{4}\right)^{2x}$

$\Rightarrow 2x = 10 \Rightarrow x = 5$

98. (d) $a^2 + b^2 + c^2 + 3$

$= 2a - 2b - 2c$

$\Rightarrow a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$

$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

$\therefore a - 1 = 0 \Rightarrow a = 1$

$b + 1 = 0 \Rightarrow b = -1$

$c + 1 = 0 \Rightarrow c = -1$

$\therefore 2a - b + c = 2 + 1 - 1 = 2$

99. (d) $\frac{x}{a} = \frac{1}{a} - \frac{1}{x}$

$\Rightarrow \frac{x}{a} = \frac{x-a}{ax}$

$\Rightarrow x^2 = x - a$

$\Rightarrow x - x^2 = a$

100. (b) $\left(x + \frac{1}{x}\right) = 4$

On squaring both sides

$x^2 + \frac{1}{x^2} + 2 = 16$

$\Rightarrow x^2 + \frac{1}{x^2} = 14$

On squaring again

$x^4 + \frac{1}{x^4} + 2 = 196$

$\Rightarrow x^4 + \frac{1}{x^4} = 194$

★★★