SSC CGL (TIER-II) Mathematics Practice Set

Answers with Explanation

1. (b) $0.9 = \frac{9}{10}; 0.\bar{9} = \frac{9}{9} = 1,$

$0.0\bar{9} = \frac{9}{90} = \frac{1}{10};$

$0.\overline{09} = \frac{9}{99} = \frac{1}{11}$

2. (a) Let number (dividend) be X.
$\therefore$ X = 296 × Q + 75 where Q is the quotient and can have the values 1, 2, 3 etc.
= 37 × 8 × Q + 37 × 2 + 1
= 37 (8Q + 2) + 1
Thus we see that the remainder is 1.
[Remark : When the second divisor is a factor of the first divisor, the second remainder is obtained by dividing the first remainder by the second divisor.
Hence, divide 75 by 37, the remainder is 1].

3. (b) By the Binomial expansion we have
$(x + 1)^n = x^n + {}^nc_1\ x^{n-1} + {}^nc_2\ x^{n-2} + ... + {}^nc_{n-1}\ x + 1$
Here, each term except the last term contains x. Obviously, each term except the last term is exactly divisible by x.
Following the same logic,
$7^{19} = (6 + 1)^{19}$ has each term except last term divisible by 6.
Hence, $7^{19} + 2$ when divided by 6 leaves remainder =1 + 2 = 3

4. (b) Expression
$= 2^{71} (1 + 2 + 4 + 8)$
$= 2^{71} \times 15 = 2^{71} \times 3 \times 5$
Which is exactly divisible by 10.

5. (a) HCF of two-prime numbers = 1
$\therefore$ Product of numbers = their LCM = 117
117 = 13 × 9 where 13 & 9 are co-prime.
L.C.M (13, 9) = 117.

6. (b) LCM of 4, 6, 10, 15 = 60
Least number of 6 digits = 100000
The least number of 6 digits which is exactly divisible by 60 = 100000 + (60 – 40) = 100020
$\therefore$ Required number (N)
= 100020 + 2 = 100022
Hence, the sum of digits = 1 + 0 + 0 + 0 + 2 + 2 = 5

7. (a) The number will be HCF of 307 – 3 = 304 and 330 – 7 = 323.

304) 323 (1
304
19) 304 (16
19
114
114
×

$\therefore$ Required number = 19

8. (a) $\frac{a^2 - b^2 - c^2 - 2bc}{a^2 + b^2 - 2ab - c^2}$

$= \frac{a^2 - (b^2 + c^2 + 2bc)}{(a^2 + b^2 - 2ab) - c^2}$

$= \frac{a^2 - (b + c)^2}{(a - b)^2 - c^2}$

$= \frac{(a + b + c)(a - b - c)}{(a - b + c)(a - b - c)}$

$= \frac{a + b + c}{a - b + c} = \frac{0.25 - 0.05 + 0.5}{0.25 + 0.05 + 0.5}$

$= \frac{0.7}{0.8} = \frac{7}{8}$

9. (a) $\frac{\frac{79}{14}}{5 + \frac{3}{3 + \frac{5}{3}}} = \frac{\frac{79}{14}}{5 + \frac{3}{\frac{9+5}{3}}}$

$= \frac{\frac{79}{14}}{5 + \frac{9}{14}} = \frac{\frac{79}{14}}{\frac{70+9}{14}}$

$= \frac{79}{14} \times \frac{14}{79} = 1$

10. (c) $\frac{\frac{5}{3} \times \frac{7}{51} \text{ of } \frac{17}{5} - \frac{1}{3}}{\frac{2}{9} \times \frac{5}{7} \text{ of } \frac{28}{5} - \frac{2}{3}} = \frac{\frac{5}{3} \times \frac{7}{15} - \frac{1}{3}}{\frac{2}{9} \times 4 - \frac{2}{3}}$

$$= \frac{\frac{7}{9}-\frac{1}{3}}{\frac{8}{9}-\frac{2}{3}} = \frac{4}{9}\times\frac{9}{2} = 2$$

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11. (b) $x^2 + \frac{1}{x^2} = 66$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 + 2 = 66$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 66-2 = 64$$

$$\Rightarrow x-\frac{1}{x} = \pm 8$$

$\therefore$ Expression $= \frac{x^2-1+2x}{x}$ অ্যাচিভার্স

$$= \frac{x^2}{x} - \frac{1}{x} + 2 = x - \frac{1}{x} + 2$$

Putting the value of $x-\frac{1}{x}$

= 8 + 2 or – 8 + 2 = 10 or –6

12. (b) $\left(\frac{1}{9}\right)^2\left\{1-9\left(\frac{16-1}{90}\right)^2\right\}$

$$= \frac{1}{81}\left\{1-\frac{9\times15\times15}{90\times90}\right\}$$

$$= \frac{1}{81}\times\left\{1-\frac{1}{4}\right\} = \frac{1}{81}\times\frac{3}{4} = \frac{1}{108}$$ অ্যাচিভার্স

13. (d) Total cost price of 20 kg of mixed rice ₹(12 × 30 + 8 × 40) = ₹680

$\therefore$ Average per kg price $= \frac{680}{20} =$ ₹ 34

14. (b) Required mean

$$= \frac{1\times1+2\times2+3\times3+4\times4+5\times5+6\times6+7\times7}{1+2+3+4+5+6+7}$$

$$= \frac{1+4+9+16+25+36+49}{28} = \frac{140}{28} = 5$$

15. (b) x + x + 1 + x + 2 + x + 3 + x + 4 + x + 5 + x + 6 + x + 7

= 6.5 × 8 = 52

$\Rightarrow$ 8x + 28 = 52

$\Rightarrow$ 8x = 52 – 28 = 24 অ্যাচিভার্স

$\Rightarrow$ x = 3

$\therefore$ Required average $= \frac{3+10}{2} = 6.5$

16. (b) Average of *a, b, c, d, e, f, g* = *d*

Average of *j, k, l, m, n,* = *l*

$\therefore$ Required average $= \frac{d+l}{2}$

17. (d) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{1}{2}$ অ্যাচিভার্স

$$\therefore \frac{3a}{3b} = \frac{5c}{5d} = \frac{7e}{7f} = \frac{1}{2}$$

$$\therefore \frac{3a+5c+7e}{3b+5d+7f} = \frac{1}{2} = 1:2$$

18. (d) According to the question,

$$A\times\frac{2}{3} = B\times\frac{75}{100} = C\times\frac{6}{10}$$

$$\Rightarrow A\times\frac{2}{3} = B\times\frac{3}{4} = C\times\frac{3}{5}$$

Now, $A\times\frac{2}{3} = B\times\frac{3}{4}$ অ্যাচিভার্স

$$\Rightarrow \frac{A}{B} = \frac{3}{4}\times\frac{3}{2} = \frac{9}{8} \Rightarrow A:B = 9:8$$

and $B\times\frac{3}{4} = C\times\frac{3}{5}$

$$\Rightarrow \frac{B}{C} = \frac{3}{5}\times\frac{4}{3} = \frac{4}{5} = \frac{8}{10}$$

= B : C = 8 : 10

$\therefore$ A : B : C = 9 : 8 : 10

19. (b) $x + \frac{1}{x} = 2$

On squaring both sides,

$x^2 + \frac{1}{x^2} + 2 = 4$ অ্যাচিভার্স

$$\Rightarrow x^2 + \frac{1}{x^2} = 4-2 = 2$$

Again, $x + \frac{1}{x} = 2$

On cubing both sides,

$$\left(x+\frac{1}{x}\right)^3 = 8$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right) = 8$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\times2 = 8$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 8-6 = 2$$ অ্যাচিভার্স

$$\therefore \left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)$$

= 2 × 2 = 4

20. (a) Let numbers = 5x and 4x

$\therefore 5x \times \frac{40}{100} = 12$

$\Rightarrow$ 2x = 12 $\Rightarrow$ x = 6 and

Second number = 6 × 4 = 24

$\therefore 50\%$ of $24 = 24 \times \frac{50}{100} = 12$

21. (a) $(A+B) \times \frac{40}{100}$

$= (A+B) \times \frac{60}{100}$

$\Rightarrow$ 2(A + B) = 3 (A – B)

$\Rightarrow$ 2A + 2B = 3A – 3B

$\Rightarrow$ A = 5B

$\therefore \frac{2A-3B}{A+B} = \frac{10B-3B}{5B+B}$

$= \frac{7B}{6B} = \frac{7}{6}$

22. (a) $a \times \frac{120}{100} = b \times \frac{80}{100}$

$\Rightarrow \frac{b}{a} = \frac{120}{80} = \frac{3}{2}$

$\therefore \frac{b+a}{b-a} = \frac{\frac{b}{a}+1}{\frac{b}{a}-1} = \frac{\frac{3}{2}+1}{\frac{3}{2}-1} = \frac{\frac{5}{2}}{\frac{1}{2}} = 5$

23. (a) $a + \frac{1}{a} = \sqrt{3}$

On squaring both sides.

$a^2 + \frac{1}{a^2} + 2 = 3$

$\Rightarrow a^2 + \frac{1}{a^2} = 3 - 2 = 1$

On cubing both sides,

$\left(a^2 + \frac{1}{a^2}\right)^3 = 1^3$

$\Rightarrow a^6 + \frac{1}{a^6} + 3\left(a^2 + \frac{1}{a^2}\right) = 1$

$\Rightarrow a^6 + \frac{1}{a^6} = 1 - 3 = -2$

$\Rightarrow \frac{a^{12}+1}{a^6} = -2$

$\Rightarrow a^6 + 2a^6 + 1 = 0$

$\Rightarrow (a^6 + 1)^2 = 0$

$\Rightarrow a^6 + 1 = 0$

$\therefore$ Expression

$= a^{18} + a^{12} + a^6 + 1$

$= a^{12}(a^6 + 1) + (a^6 + 1) = 0$

24. (c) Required percentage

$= \frac{x}{100-x} \times 100$

$= \frac{40}{60} \times 100 = \frac{200}{3} = 66.66\%$

25. (d) If the cost price be ₹x, then

$S.P. = \frac{100}{95}x = ₹\ \frac{20}{19}x$

$\therefore$ Gain $= \frac{20x}{19} - x = ₹\ \frac{x}{19}$

$\therefore$ Gain percent $= \frac{\frac{x}{19}}{x} \times 100 = 5.26\%$

26. (c) Let the cost price of one table = x

$\therefore$ Cost price of 15 tables = 15x

and cost price of 20 tables = 20x

According to the question

Selling price of 20 tables

= cost price of 15 tables = 15x

$\therefore$ Loss = 20x – 15x = 5x

$\therefore$ Loss% $= \frac{5x \times 100}{20x} = 25\%$

27. (b) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$\Rightarrow 9 = 3(a^2 + b^2 - ab)$

$\Rightarrow a^2 + b^2 - ab = \frac{9}{3} = 3$

$\Rightarrow (a + b)^2 - 2ab - ab = 3$

$\Rightarrow 9 - 3ab = 3$

$\Rightarrow 3ab = 9 - 3 = 6$

$\Rightarrow ab = 2$

$\therefore \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{3}{2}$

28. (c) Let the CP of each article be ₹1

$\therefore$ CP of 9 articles = ₹9

$\therefore$ SP of 9 articles = ₹8

$\therefore$ Loss = ₹1

$\therefore$ Loss% $= \frac{1}{9} \times 100$

$= \frac{100}{9} = 11\frac{1}{9}\%$

29. (d) Let Ravi buy 10 toffees.

$\therefore$ C.P. = ₹5

S.P. = ₹2

$\therefore$ Loss% $= \frac{5-2}{5} \times 100 = 60\%$

30. (c) Equivalent discount

$= 10 + 5 - \frac{10 \times 5}{100} = 14.5\%$

$\therefore$ CP (for buyer)

= 85.5% of ₹200000

$= ₹\left(\frac{855 \times 200000}{100}\right) = ₹171000$

SP = ₹179550

Gain = ₹(179550 –171000) = ₹8550

$\therefore$ Gain % $= \frac{8550}{171000} \times 100 = 5\%$

31. (b) Single equivalent discount for 10% and 12%.

$= \left(12 + 10 - \frac{12 \times 10}{100}\right)\% = 20.8\%$

Single equivalent discount for 20.8% and 5%.

$= \left(20.8 + 5 - \frac{20.8 \times 5}{100}\right)\% = 24.76\%$

32. (d) Single equivalent discount for consecutive discounts of x% and y%

$= \left(x + y - \frac{xy}{100}\right)\%$

33. (c) $4a - \frac{4}{a} = -3$

On dividing by 4,

$\Rightarrow a - \frac{1}{a} = \frac{-3}{4}$

$\therefore a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3a \times \frac{1}{a}\left(a - \frac{1}{a}\right)$

$= \left(\frac{-3}{4}\right)^3 + 3 \times \frac{-3}{4}$

$= -\frac{27}{64} - \frac{9}{4} = \frac{-27-144}{64} = \frac{-171}{64}$

$\therefore a^3 - \frac{1}{a^3} + 3 = \frac{-171}{64} + 3$

$= \frac{-171+192}{64} = \frac{21}{64}$

34. (c) C.P. of article = ₹100

and marked price of article = ₹x (let)

$\therefore x \times \frac{90}{100} = 117$

$\Rightarrow x = \frac{117 \times 100}{90} = ₹130$

i.e. 30% above the cost price

35. (c) Simple interest gained from ₹500

$= \frac{500 \times 12 \times 4}{100} = ₹240$

Let the other Principal be x.

S.I. gained = ₹(480 – 240) = ₹240

$\therefore \frac{x \times 10 \times 4}{100} = 240$

$\Rightarrow x = \frac{240 \times 100}{40} = ₹600$

36. (c) **Case I :**

$S.I. = \frac{5000 \times 2 \times 4}{100} = ₹400$

Case II :

$S.I. = \frac{5000 \times 25 \times 2}{100 \times 4} = ₹625$

$\therefore$ Gain = ₹(625 – 400) = ₹225

37. (a) Principal $= \frac{S.I. \times 100}{Time \times Rate} = \frac{60 \times 100}{5 \times 6} = Rs.\ 200$

38. (d) $x + \frac{1}{x} = 3$

On squaring both sides,

$x^2 + \frac{1}{x^2} + 2 = 9$

$\Rightarrow x^2 + \frac{1}{x^2}$

= 9 – 2 = 7 ... (i)

Again,

$\left(x + \frac{1}{x}\right)^3 = 3^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$

$\therefore x^3 + \frac{1}{x^3} = 27 - 9 = 18$ ... (ii)

$\therefore \left(x^3 - \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right)$

= 18 × 7 = 126

$\Rightarrow x^5 + x + \frac{1}{x^5} + \frac{1}{x} = 126$

$\Rightarrow x^5 + \frac{1}{x^5} = 126 - 3 = 123$

39. (d) Let principal be Rs. x.
∴ Amount = Rs. 2x
Interest = Rs. (2x – x) = Rs. x

$$\therefore \text{Rate} = \frac{\text{S.I.} \times 100}{\text{Principal} \times \text{Time}}$$

$$= \frac{x \times 100}{x \times 8} = \frac{25}{2} = 12\frac{1}{2}\% \text{ per annum}$$

40. (c) Let r_1, and r_2 be the required rate of interest

Then, $13.50 = \frac{1500 \times 3 \times r_1}{100} - \frac{1500 \times r_2}{100}$

$$= \frac{4500}{100}(r_1 - r_2)$$

$$r_1 - r_2 = \frac{135}{450} = \frac{27}{90} = \frac{3}{10} = 0.3\%$$

41. (b) Let the sum be ₹ P and rate of interest be R% per annum. Then,

$$P\left(1+\frac{R}{100}\right)^2 = 9680 \quad \text{... (i)}$$

$$P\left(1+\frac{R}{100}\right)^3 = 10648 \quad \text{... (ii)}$$

On dividing equation (ii) by (i)

$$1+\frac{R}{100} = \frac{10648}{9680}$$

$$\Rightarrow \frac{R}{100} = \frac{10648}{9680} - 1$$

$$= \frac{10648 - 9680}{9680}$$

$$\Rightarrow \frac{R}{100} = \frac{968}{9680} = \frac{1}{10}$$

$$\Rightarrow R = \frac{1}{10} \times 100 = 10\%$$

42. (a) $A = P\left(1+\frac{R}{100}\right)^T$

Let rate be 'r'

$$\Rightarrow \frac{1102.50}{1000} = \left(1+\frac{r}{100}\right)^2$$

$$\Rightarrow \frac{11025}{10000} = \left(1+\frac{r}{100}\right)^2$$

$$\Rightarrow \left(\frac{105}{100}\right)^2 = \left(1+\frac{r}{100}\right)^2$$

$$\Rightarrow 1+\frac{r}{100} = \frac{105}{100}$$

$\Rightarrow$ r = 5%

43. (c) Principal (P) = Rs. S
Rate (R) = 2r% per annum

$$\therefore \text{Amount} = P\left(1+\frac{R}{100}\right)^T$$

$$= S\left(1+\frac{2r}{100}\right)^3 = S\left(1+\frac{r}{50}\right)^3$$

44. (b) $2x + \frac{2}{x} = 3$

On dividing by 2,

$$x + \frac{1}{x} = \frac{3}{2}$$

On cubing both sides,

$$\left(x+\frac{1}{x}\right)^3 = \left(\frac{3}{2}\right)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right) = \frac{27}{8}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + \frac{3\times 3}{2} = \frac{27}{8}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2}$$

$$= \frac{27-36}{8}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = \frac{-9}{8}$$

$$\therefore x^3 + \frac{1}{x^3} + 2$$

$$= 2 - \frac{9}{8} = \frac{16-9}{8} \quad \frac{7}{8}$$

45. (d) $\text{Principal} = \frac{\text{S.I.} \times 100}{\text{Time} \times \text{Rate}}$

$$= \frac{80 \times 100}{2 \times 4} = ₹\,1000$$

$$\therefore \text{C.I.} = P\left[\left(1+\frac{R}{100}\right)^T - 1\right]$$

$$= 1000\left[\left(1+\frac{4}{100}\right)^2 - 1\right]$$

$$= 1000\left[\left(\frac{26}{25}\right)^2 - 1\right]$$

$$= 1000\left(\frac{676}{625} - 1\right)$$

$= 1000\left(\frac{676-625}{625}\right)$

$= \frac{1000\times 51}{625} = ₹\ 81.60$ অ্যাচিভার্স

46. (d) S.I. $= ₹\frac{2500\times 2\times 4}{100} = ₹\ 200$

C.I. $= ₹\ 2500\left[\left(1+\frac{4}{100}\right)^2 - 1\right]$

$= ₹ 2500\left[\left(\frac{26}{25}\right)^2 - 1\right]$

$= ₹\ \frac{(676-625)}{625}\times 2500$ অ্যাচিভার্স

$= ₹\ \frac{51}{625}\times 2500 = ₹ 204$

∴ The required difference

= C.I. – S.I. = ₹(204 – 200) = ₹4

47. (a) (A + B)'s 1 day's work $= \frac{1}{15}$

B's 1 day's work $= \frac{1}{20}$

∴ A's 1 day's work

$= \frac{1}{5} - \frac{1}{20} = \frac{4-3}{60} = \frac{1}{60}$

∴ A alone will do the work in 60 days.

48. (d) A's 1 day's work $= \frac{1}{m}$ অ্যাচিভার্স

B's 1 day's work $= \frac{1}{n}$

∴ (A + B)'s 1 day's work $= \frac{1}{m} + \frac{1}{n}$

$= \frac{n+m}{mn} = \frac{m+n}{mn}$

∴ Required time $= \frac{mn}{m+n}$

49. (c) (A + B)'s 1 day's work

$= \frac{1}{25} + \frac{1}{30} = \frac{6+5}{150} = \frac{11}{150}$ অ্যাচিভার্স

∴ (A + B)'s 5 days' work $= \frac{5\times 11}{150} = \frac{11}{30}$

∴ Remaining work $= 1 - \frac{11}{30} = \frac{30-11}{30} = \frac{19}{30}$

50. (a) $\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)} + \frac{1}{(c+a)(a+b)}$

$= \frac{c+a+a+b+b+c}{(a+b)(b+c)(c+a)}$

$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)} = 0$ অ্যাচিভার্স

51. (d) Let the work be finished in x days.

According to the question,

A worked for x days while B worked for (x – 3) days

$\therefore \frac{x}{18} + \frac{x-3}{12} = 1$

$\Rightarrow \frac{2x+3x-9}{36} = 1$

$\Rightarrow$ 5x – 9 = 36

$\Rightarrow$ 5x = 45 অ্যাচিভার্স

$\Rightarrow x = \frac{45}{5} = 9$ days.

Hence, the work was completed in 9 days.

52. (d) Part of cistern filled by three pipes in an hour

$= \frac{1}{3} + \frac{1}{5} - \frac{1}{2} = \frac{10+6-15}{30} = \frac{1}{30}$

Hence, the cistern will be filled in 30 hours.

53. (b) Let the capacity of the tank be x litres then

$\frac{x}{3} = 80$

∴ x = 240

$\therefore \frac{x}{2} = \frac{240}{2} = 120$ litres অ্যাচিভার্স

54. (d) Pipe A fills the tank in $\frac{72}{5}$ minutes.

∴ Part of the tank filled by A in 30 minutes

$= \frac{2}{75}\times 30 = \frac{4}{5}$

Remaining part $= 1 - \frac{4}{5} = \frac{1}{5}$

Now, 1 part is filled by pipe B in 45 minutes

$\therefore \frac{1}{5}$ part is filled in $= 45\times\frac{1}{5} = 9$ minutes

Hence, the pipe B should be turned off after 9 minutes. অ্যাচিভার্স

55. (d) Difference of time = 4.30 p.m – 11.a.m.

$= 5\frac{1}{2}$ hours $= \frac{11}{2}$ hours

Distance covered in $\frac{11}{2}$ hours

$= \frac{5}{6} - \frac{3}{8} = \frac{20-9}{24} = \frac{11}{24}$ part

$\because \frac{11}{24}$ part of the journey is covered in $\frac{11}{2}$ hours

$\Rightarrow \frac{3}{8}$ part of the journey is covered in

$= \frac{11}{2} \times \frac{24}{11} \times \frac{3}{8} = \frac{9}{2}$ hours $= 4\frac{1}{2}$ hours

Clearly the person started at 6.30 a.m.

56. (b) Let the total journey be of x km, then

$$\frac{2x}{15} + \frac{9x}{20} + 10 = x$$

$$\Rightarrow x - \frac{2x}{15} - \frac{9x}{20} = 10$$

$$\Rightarrow \frac{60x - 8x - 27x}{60} = 10$$

$$\Rightarrow \frac{25x}{60} = 10$$

$$\Rightarrow x = \frac{60 \times 10}{25} = 24 \text{ km}$$

57. (a) $a + \frac{1}{a} = -2$

$\Rightarrow a^2 + 1 = -2a$

$\Rightarrow a^2 + 2a + 1 = 0$

$\Rightarrow (a + 1)^2 = 0$

$\Rightarrow a + 1 = 0$

$\Rightarrow a = -1$

$\therefore (a)^{1000} + (a)^{-1000}$

$= (-1)^{1000} + (-1)^{-1000}$

$= 1 + 1 = 2$

58. (b) When a train crosses a tunnel, it covers a distance equal to the sum of its own length and tunnel.

Let the length of tunnel be x Speed = 78 kmph

$= \frac{78 \times 1000}{60 \times 60} \text{ m/sec.} = \frac{65}{3} \text{ m/sec.}$

$\therefore \text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$$\Rightarrow \frac{65}{3} = \frac{800 + x}{60}$$

$\Rightarrow (800 + x) \times 3 = 65 \times 60$

$\Rightarrow 800 + x = 65 \times 20$ m

$\Rightarrow x = 1300 - 800 = 500$

$\therefore$ Length of tunnel = 500 metres.

59. (c) Let the speed of man in still water be x kmph and rate of stream be y kmph

$\therefore$ Distance $= \frac{750}{1000} \text{ km} = \frac{3}{4} \text{ km.}$

Time = 675 seconds

$= \frac{675}{60} = 11\frac{1}{4}$ minutes

$$\therefore x - y = \frac{\frac{3}{4}}{\frac{45}{4}} = \frac{3}{45} = \frac{1}{15} \text{ km/min}$$

and $x + y = \frac{\frac{3}{4}}{\frac{15}{2}}$

$= \frac{3}{4} \times \frac{2}{15} = \frac{1}{10}$ km/min

$\therefore$ Speed in still water

$= \frac{1}{2}\left(\frac{1}{10} + \frac{1}{15}\right) = \frac{1}{2}\left(\frac{3+2}{30}\right) = \frac{1}{12}$ km/min

$= \frac{1}{12} \times 60$ kmph = 5 kmph

60. (a) Let the speed of the current be x kmph. According to the question,

$$\frac{6}{6 - x} = 3$$

$\Rightarrow 18 - 3x = 6 \Rightarrow 3x = 18 - 6$

$\Rightarrow x = \frac{12}{3} = 4$ kmph

61. (c) $\frac{x^3 + 3y^2x}{y^3 + 3x^2y} = \frac{35}{19}$

By componendo and dividendo,

$$\frac{x^3 + 3y^2x + y^3 + 3x^2y}{x^3 + 3y^2x - y^3 - 3x^2y}$$

$$= \frac{35 + 19}{35 - 19} = \frac{54}{16}$$

$$\Rightarrow \frac{(x + y)^3}{(x - y)^3} = \frac{27}{8} = \left(\frac{3}{2}\right)^3$$

$$\Rightarrow \frac{x + y}{x - y} = \frac{3}{2}$$

By componendo and dividendo again

$$\frac{x + y + x - y}{x + y - x + y} = \frac{3 + 2}{3 - 2} \Rightarrow \frac{x}{y} = 5$$

62. (c) Let the distance of the destination from the starting point be x km.

Rate downstream = (10 + 4) kmph = 14 kmph

Rate upstream = (10 – 4) kmph = 6 kmph

According to the question,

$\frac{x}{14}+\frac{x}{6}=5$

$\Rightarrow \frac{3x+7x}{42}=5$

$\Rightarrow 10x = 42 \times 5$

$\Rightarrow x=\frac{42\times5}{10}=21$ km.

63. (c) $\ldots\ldots^{x} = 3125$

$\Rightarrow (53)^x = 5^5 \Rightarrow 5^{3x} = 5^5$

$\Rightarrow 3x = 5$

$\Rightarrow x=\frac{5}{3}$

64. (c) $2^{2x-y} = 16 = 2^4$

$\Rightarrow 2x - y = 4$ ..... (i)

$2^{x+y} = 32 = 2^5$

$\Rightarrow x + y = 5$ ..... (ii)

On adding equations (i) and (ii),

$3x = 9 \Rightarrow x = 3$

From equation (ii),

$y = 5 - x = 5 - 3 = 2$

$\therefore xy = 3 \times 2 = 6$

65. (b) $\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}-\sqrt{3-x}}=\frac{2}{1}$

By componendo and dividendo,

$\Rightarrow \frac{2\sqrt{3+x}}{2\sqrt{3-x}}=\frac{2+1}{2-1}=3$

Squaring on both sides, we get

$\frac{3+x}{3-x}=9$

$\Rightarrow 3 + x = 27 - 9x$

$\Rightarrow 9x + x = 27 - 3 = 24$

$\Rightarrow x=\frac{24}{10}=\frac{12}{5}$

66. (a) $\sqrt{1+\frac{x}{961}}=\frac{32}{31}$

Squaring both sides,

$1+\frac{x}{961}=\left(\frac{32}{31}\right)^2=\frac{1024}{961}$

$\Rightarrow \frac{x}{961}=\frac{1024}{961}-1=\frac{1024-961}{961}=\frac{63}{961}$

$\Rightarrow x = 63$

67. (c) $x+\frac{1}{x}=99$

$\therefore \frac{100x}{2x^2+102x+2}$

$=\frac{100x}{2x^2+2+102x}$

On dividing by x,

$=\frac{100}{2x+\frac{2}{x}+102}$

$=\frac{100}{2\left(x+\frac{1}{x}\right)+102}$

$=\frac{100}{2\times99+102}=\frac{100}{300}=\frac{1}{3}$

68. (a) $x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

Dividing both sides by x,

$\Rightarrow x+\frac{1}{x}=3$

$\therefore x^2+x+\frac{1}{x}+\frac{1}{x^2}$

$=\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)$

$=\left(x+\frac{1}{x}\right)^2-2+\left(x+\frac{1}{x}\right)$

$= 9 - 2 + 3 = 10$

69. (c) Amount spent on cement

$=\left(\frac{100}{360}\times72\right)\%$ of 600000

$=\frac{20}{100}\times600000 = ₹\,1,20,000$

70. (b) Difference (%)

$=\left(\frac{100}{360}\times90\right)\%-\left(\frac{100}{360}\times54\right)\%$

$= 25\% - 15\%$

$= 10\%$ of the total cost

71. (c) Required per cent

$=\frac{100}{360^\circ}\times(72^\circ+54^\circ+54^\circ)$

$=\frac{100}{360^\circ}\times180^\circ = 50$ per cent

72. (d) Differences in percentage

$=\left(\frac{100}{360^\circ}\times90^\circ\right)-\left(\frac{100}{360^\circ}\times54^\circ\right)\%$

$\therefore$ Required difference

$=600000\times\frac{10}{100} = ₹\,60,000$

73. (d) Corresponding angle of labour = 90°

Clearly, $90° \equiv \frac{1500000 \times 90}{360} = ₹\ 375000$

74. (c) $2x = \sqrt{a} + \frac{1}{\sqrt{a}}$ অ্যাচিভার্স

On squaring both sides,

$4x^2 = a + \frac{1}{a} + 2$

$\Rightarrow 4x^2 - 4 = a + \frac{1}{a} + 2 - 4$

$= a + \frac{1}{a} - 2$

$\therefore \sqrt{4x^2 - 4} = \sqrt{\left(\sqrt{a} - \frac{1}{\sqrt{a}}\right)^2}$ অ্যাচিভার্স

$= \sqrt{a} - \frac{1}{\sqrt{a}}$

$\therefore \sqrt{x^2 - 1} = \frac{1}{2}\left(\sqrt{a} - \frac{1}{\sqrt{a}}\right)$

$\therefore$ Expression $= \frac{\sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}}$

$= \frac{\frac{1}{2}\left(\sqrt{a} - \frac{1}{\sqrt{a}}\right)}{\frac{1}{2}\left(\sqrt{a} + \frac{1}{\sqrt{a}}\right) - \frac{1}{2}\left(\sqrt{a} - \frac{1}{\sqrt{a}}\right)}$ অ্যাচিভার্স

$= \frac{\frac{1}{2}\left(\sqrt{a} + \frac{1}{\sqrt{a}}\right)}{\frac{1}{\sqrt{a}}} = \frac{1}{2}\sqrt{a}\left(\sqrt{a} - \frac{1}{\sqrt{a}}\right)$

$= \frac{1}{2}(a - 1)$

75. (c) $x = \frac{1}{2 + \sqrt{3}}$

$= \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{2 - \sqrt{3}}{4 - 3}$

$= 2 - \sqrt{3}$

$\therefore y = \frac{1}{2 - \sqrt{3}} = 2 + \sqrt{3}$ অ্যাচিভার্স

$\therefore x + y = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

$xy = (2 - \sqrt{3})(2 + \sqrt{3})$

$= 4 - 3 = 1$

$\therefore$ $8xy\ (x^2 + y^2)$

$= 8xy\ [(x + y)^2 - 2xy]$

$= 8 \times 1\ (4^2 - 2 \times 1)$

$= 8\ (16 - 2) = 8 \times 14 = 112$

76. (d) $2x + \frac{1}{3x} = 5$ অ্যাচিভার্স

$\Rightarrow \frac{6x^2 + 1}{3x} = 5$

$\Rightarrow 6x^2 + 1 = 15x$

$\therefore \frac{5x}{6x^2 + 20x + 1}$

$= \frac{5x}{6x^2 + 1 + 20x}$

$= \frac{5x}{15x + 20x} = \frac{5x}{35x} = \frac{1}{7}$

77. (a) $a + \frac{1}{a} + 1 = 0$ অ্যাচিভার্স

$\Rightarrow a^2 + a + 1 = 0$

$\Rightarrow a^4 - a = a(a^3 - 1)$

$= a(a - 1)(a^2 + a + 1) = 0$

78. (b) $(a^2 + b^2)^3 = (a^3 + b^3)^2$

$\Rightarrow a^6 + b^6 + 3a^2b^2(a^2 + b^2)$

$= a^6 + b^6 + 2a^3b^3$

$\Rightarrow 3(a^2 + b^2) = 2ab$

$\Rightarrow \frac{a^2 + b^2}{ab} = \frac{2}{3}$

$\Rightarrow \frac{a}{b} + \frac{b}{a} = \frac{2}{3}$ অ্যাচিভার্স

79. (a) $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1}$

$= a\sqrt[3]{4} + b\sqrt[3]{2} + c$

$\Rightarrow \frac{1}{2^{\frac{2}{3}} + 2^{\frac{1}{3}} + 1}$

$= a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$

$\Rightarrow \frac{\left(2^{\frac{1}{3}} - 1\right)}{\left(2^{\frac{1}{3}} - 1\right)\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}} + 1\right)}$ অ্যাচিভার্স

$= a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$

$\Rightarrow \frac{2^{\frac{1}{3}} - 1}{2 - 1} = a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$

$[\because (a-b)(a^2+ab+b^2) = a^3-b^3]$
$\Rightarrow a = 0, b = 1, c = -1$
$\therefore a + b + c = 0 + 1 - 1 = 0$

80. (a) $\sec(7\theta + 28°)$
$= \text{cosec}(30° - 3\theta)$
$\Rightarrow \sec(7\theta + 28°)$
$= \sec(90° - (30° - 3\theta))$
$\Rightarrow 7\theta + 28° = 90° - 30° + 3\theta$
$\Rightarrow 4\theta = 90° - 30° - 28° = 32°$
$\Rightarrow \theta = \frac{32^o}{4} = 8^o$

81. (c) $\text{cosec}\theta - \cot\theta = \frac{7}{2}$ .....(i)
$\text{cosec}^2\theta - \cot^2\theta = 1$
$\Rightarrow (\text{cosec}\theta + \cot\theta)(\text{cosec}\theta - \cot\theta) = 1$
$\Rightarrow \text{cosec}\theta + \cot\theta$
$= \frac{1}{\text{cosec}\theta - \cot\theta} = \frac{2}{7}$ .....(ii)
On adding both equations,
$2\text{cosec}\theta = \frac{7}{2} + \frac{2}{7}$
$= \frac{49+4}{14} = \frac{53}{14}$
$\Rightarrow \text{cosec}\theta = \frac{53}{28}$

82. (d) $2(\cos^2\theta - \sin^2\theta) = 1$
$\Rightarrow \cos^2\theta - (1 - \cos^2\theta) = \frac{1}{2}$
$\Rightarrow 2\cos^2\theta = 1 + \frac{1}{2} = \frac{3}{2}$
$\Rightarrow \cos^2\theta = \frac{3}{4}$
$\Rightarrow \sec^2\theta = \frac{4}{3}$
$\Rightarrow 1 + \tan^2\theta = \frac{4}{3}$
$\Rightarrow \tan^2\theta = \frac{4}{3} - 1 = \frac{1}{3}$
$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \cot\theta = \sqrt{3}$

83. (a) $\tan\theta = 1 \Rightarrow \theta = 45°$
$\therefore \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$
$= \frac{8\times\frac{1}{\sqrt{2}} + \frac{5}{\sqrt{2}}}{\frac{1}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{7}{\sqrt{2}}}$
$= \frac{\frac{13}{\sqrt{2}}}{\frac{13}{2\sqrt{2}}} = \frac{13}{\sqrt{2}} \times \frac{2\sqrt{2}}{13} = 2$

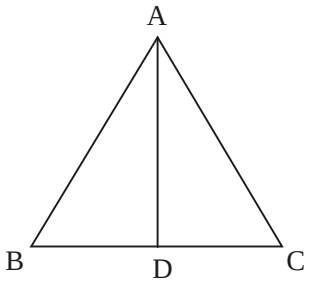
84. (b) $a^3 + \frac{1}{a^3} = 2$
$\Rightarrow a^6 + 1 = 2a^3$
$\Rightarrow a^6 - 2a^3 + 1 = 0$
$\Rightarrow (a^3 - 1)^2 = 0$
$\Rightarrow a^3 - 1 = 0$
$\Rightarrow a^3 = 1 \Rightarrow a = 1$
$\therefore \frac{a^2+1}{a} = 1 + 1 = 2$

85. (c) $\tan 2\theta = \frac{1}{\tan 4\theta} = \cot 4\theta$
$\Rightarrow \tan 2\theta = \tan(90° - 4\theta)$
$\Rightarrow 2\theta = 90° - 4\theta$
$\Rightarrow 6\theta = 90° \Rightarrow \theta = 15°$
$\therefore \tan 3\theta = \tan 45° = 1$

86. (c) $(l^2.m^2)(l^2 + m^2 + 3)$
$= (\text{cosec}\ \theta - \sin\theta)^2$
$(\sec\theta - \cos\theta)^2$
$\{(\text{cosec}\ \theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3\}$
$= \left(\frac{1}{\sin\theta} - \sin\theta\right)^2 \left(\frac{1}{\cos\theta} - \cos\theta\right)^2$
$\left\{\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 + 3\right\}$
$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2$
$\left\{\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2 + 3\right\}$
$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^2 \left(\frac{\sin^2\theta}{\cos\theta}\right)^2$
$\left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right\}$
$= \frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^4\theta}{\cos^2\theta} \left\{\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4}{\cos^2\theta} + 3\right\}$
$= \cos^2\theta \times \sin^2\theta$
$\left\{\frac{\cos^6\theta + \sin^6\theta + 3\cos^2\theta.\sin^2\theta}{\cos^2\theta.\sin^2\theta}\right\}$

$= \cos^6\theta + \sin^6\theta + 3\cos^2\theta.\sin^2\theta$
$= \{(\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta.\sin^2\theta$
$(\cos^2\theta + \sin^2\theta)\} + 3\cos^2\theta.\sin^2\theta$
$[\because a^3 + b^3 = (a + b)^3 - 3ab(a + b)]$
$= 1 - 3\cos^2\theta.\sin^2\theta + 3\cos^2\theta.\sin^2\theta = 1$

87. (b)



D, is the mid–point of BC.
AB = AC = 10 cm.
AD ⊥ BC
From ΔABD,

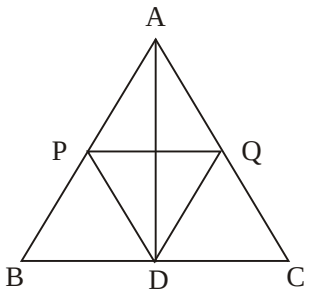
$BD = \sqrt{AB^2 - AD^2}$

$= \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$

$= \sqrt{36} = 6$ cm.

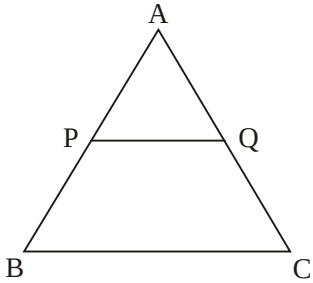
∴ BC = 2 BD = 2 × 6 = 12 cm.

88. (b)



AB = AC
Point D is the mid-point of side BC.
∴ ∠ADB = 90° = ∠ADC
PD is internal bisector of ∠ADB.
∴ ∠PDA = 45°
PQ ∥ BC
∴ ∠ADQ = 45°
∴ PDQ = 45° + 45° = 90°

89. (b)



$\frac{AP}{PB} = \frac{AQ}{QC} = \frac{1}{2}$

$\Rightarrow \frac{QC}{AQ} = \frac{2}{1} \Rightarrow \frac{QC + AQ}{AQ} = \frac{3}{1}$

⇒ AC = 3AQ = 9 cm

90. (c) $\left(a + \frac{1}{a}\right)^2 = 3$

$\Rightarrow a + \frac{1}{a} = \sqrt{3}$

On cubing both sides,

$\left(a + \frac{1}{a}\right)^3 = \left(\sqrt{3}\right)^3$

$\Rightarrow a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = 3\sqrt{3}$

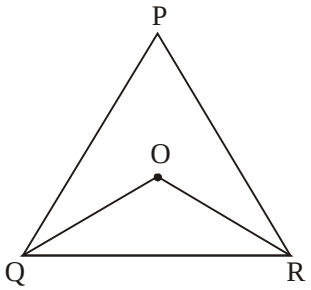
$\Rightarrow a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} = 3\sqrt{3} - 3\sqrt{3} = 0$

$\therefore a^6 - \frac{1}{a^6}$

$= \left(a^3 + \frac{1}{a^3}\right)\left(a^3 - \frac{1}{a^3}\right) = 0$

91. (d)



∠QPR = 50°
∴ ∠PQR + ∠PRQ
= 180° − 50° = 130°

$\therefore \frac{1}{2}\angle PQR + \frac{1}{2}\angle PQR = 65^\circ$

The point of intersection of internal bisectors of angles is in-centre.

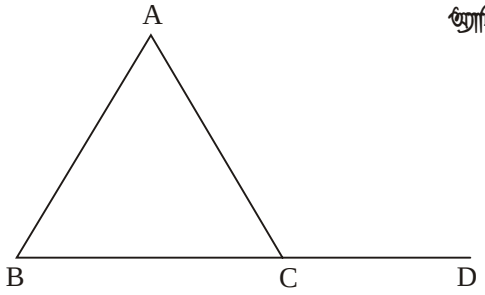
$\therefore \angle OQR = \frac{1}{2}\angle PQR$

$\angle OQR = \frac{1}{2}\angle PQR;$

In ΔOQR,

$\angle OQR + \angle QOR + \angle ORQ = 180°$
$\Rightarrow \angle QOR = 180° - 65° = 115°$

92. (c)



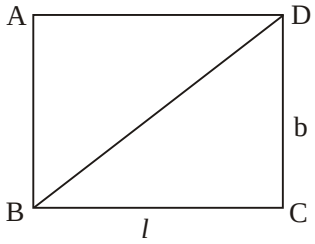
Exterior $\angle ACD$
= Interior ($\angle BAC + \angle ABC$)

$\Rightarrow 120^o = \angle A + \frac{\angle A}{2}$

$\Rightarrow 240 = 2\angle A + \angle A$
$\Rightarrow 3\angle A = 240°$

$\Rightarrow \angle A = \frac{240^o}{3} = 80^o$

93. (d)



BD = length of diagonal
= speed × time

$= \frac{52}{60} \times 15 = 13$ metre

$BD = \sqrt{l^2 + b^2}$
$\Rightarrow l^2 + b^2 = 169$ ...(i)
Again,

$(l + b) = \frac{68}{60} \times 15 = 17$ ...(ii)

$\therefore (l + b)^2 = l^2 + b^2 + 2lb$
$\Rightarrow 17^2 = 169 + 2lb$
$\Rightarrow 2lb = 289 - 169 = 120$

$\Rightarrow lb = \frac{120}{2} = 60\ m^2$

94. (b) Let the sides of parallelogram be 5x and 4x.
Base × Height = Area of parallelogram
$\therefore 5x \times 20 = 1000$

$\Rightarrow x = \frac{1000}{5 \times 20} = 10$
$\Rightarrow$ Sides = 50 and 40 units
$\therefore 40 \times h = 1000$

$\Rightarrow h = \frac{1000}{40} = 25$ units

95. (d) $x^4 + \frac{1}{x^4} = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 119 + 2 = 121$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 11^2$

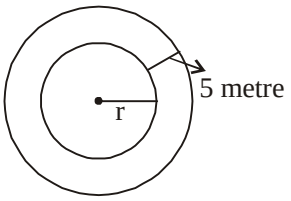
$\Rightarrow x^2 + \frac{1}{x^2} = 11$

$\therefore \left(x - \frac{1}{x}\right)^2 + 2 = 11$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 11 - 2 = 9 = 3^2$

$\Rightarrow x - \frac{1}{x} = 3$

96. (d)



More distance, more time (speed is constant)

$\therefore \frac{2\pi(r+5)}{2\pi r} = \frac{20}{19}$

$\Rightarrow \frac{r+5}{r} = \frac{20}{19}$
$\Rightarrow 20r = 19r + 95$
$\Rightarrow 20r - 19r = 95$
$\Rightarrow r = 95$ metre
$\therefore$ Internal diameter = (2 × 95) metre
= 190 metre

97. (c) Total population of village X = x (let).

$\therefore \frac{x \times 38}{100} - 12160$

$\Rightarrow x = \frac{12160 \times 100}{38} = 32000$
$\because 16\% \equiv 32000$
$\therefore 11\% \equiv \frac{32000}{16} \times 11 = 22000$

98. (c) Required ratio = 46 × 21 : 42 × 11 = 23 : 11

99. (b) Population percentage of village R = 16%
Population percentage of village Y = 15%
$\because$ 16% $\equiv$ 32000
$\therefore 15\% \equiv \frac{32000}{16} \times 15 = 32000$
$\therefore$ Population below poverty line in village Y = 52% of 30000
$= 30000 \times \frac{52}{100} = 15600$

100. (b) Population percentage of village V in 2009 = 10%
$\because$ 15% $\equiv$ 30000
$\therefore 10\% \equiv \frac{30000}{15} \times 10 = 20000$
Population of village V in 2010
$= \frac{20000 \times 110}{100} = 22000$
$\therefore$ Population below poverty line
$= \frac{22000 \times 58}{100} = 12760$

★★★