SSC CGL Tier-II Exam. Practice Set

Answers with Explanation

1. (a) $3.3^{33} \times 2.2^{33} = 6^x \Rightarrow 6.6^{33} = 6^x$
$\Rightarrow 6^{34} = 6^x \Rightarrow x = 34$

2. (c) $1 + (2-1)(2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)$
$= 1 + (2^2-1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)$
$= 1+ (2^4-1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)$
$= 1+ (2^8-1)(2^8+1)(2^{16}+1)(2^{32}+1)$
$= 1+ (2^{16}-1)(2^{16}+1)(2^{32}+1)$
$= 1+ (2^{32}-1)(2^{32}+1)$
$= 1 + 2^{64} - 1 = 2^{64}$

3. (a) xy) xy xy xy (10101
xy
xy
xy
xy

4. (a) $35^7 \times 42^5 \times 11^{21} = 5^7 \times 7^7 \times 2^5 \times 3^5 \times 7^5 \times 11^{21}$
No. of prime factors
$\rightarrow 7 + 7 + 5 + 5 + 5 + 21 = 50$

5. (c) $4^{61} + 4^{62} + 4^{63} + 4^{64} + 4^{65}$
$= 4^{61}(1 + 4 + 4^2 + 4^3 + 4^4)$
$= 4^{61} \times 1 \times \frac{4^5-1}{4-1} = 4^{61} \times \frac{1023}{3}$
$= 4^{61} \times 341 = 4^{61} \times 11 \times 31$

6. (a) 13) 55555... (427...
52
35
26
95
91

7. (a) 9 9999 | 316
9
61 | 99
61
626 | 3899
3756
143
99999 – 143 = 99856

8. (c) Let, he gave x correct answers.
$2x - (84-x)\frac{2}{3} = 144 \Rightarrow x = 75$

9. (c) $3^x \times 8^y \rightarrow 8$
$3^x \times 8^{23} \rightarrow 8$
Unit digit of 8^{23} is 2
If x = 26 then $3^{26} \rightarrow 3^2 \rightarrow 9$
$9 \times 2 = 18$

10. (c) $128 = 2^7$
In the product there is so many 5 but there is 7 twos. Then no. of zero = 7.

11. (b) $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right).....\left(1-\frac{1}{n}\right)$
$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \ldots \frac{n-1}{n} = \frac{2}{n}$

12. (b) 200 – 3 = 197
7) 197 (28
14
57
56
1

13. (b) Age of the two women
= (20 + 24 + 16) = 60
Average age $= \frac{1}{2} \times 60 = 30$ years

14. (d) Let, the number of students be x and average weight be y kg.
$\frac{xy+50}{x+1} = y+1$
$\Rightarrow xy + 50 = xy + x + y + 1 \Rightarrow x + y = 49$
$\frac{xy+50+50}{x+2} = y + \frac{3}{2}$
$\Rightarrow xy + 100 = xy + 2y + \frac{3}{2}x + 3$
$\Rightarrow 3x + 4y = 194$
$3x+4y=194$
$3x+3y=147$
$y = 47$
∴ Original average weight = 47 kg

15. (b) Let, there be 3m boys and m girls
Total score = 4mA
Total score of the boys = 3m (A+1)
Total score of the girls
= 4mA – (3mA + 3m) = mA – 3m
= m (A – 3)
$\therefore$ Average $= \frac{m(A-3)}{m} = (A-3)$

16.(a) Failed in English = (100 – 70) = 30%
Failed in Maths = (100 – 65) = 35%
Failed in both subjects = 27%
So, passed in both the subjects
= {100 – (30 + 35 – 27)} = (100 – 38)% = 62%
Hence, 62% = 248

$100\% = \frac{248}{62}\times 100 = 400$

17.(c) $x\left(1+\frac{r}{100}\right)^2 = 2x$

or, $\left(1+\frac{r}{100}\right)^2 = 2 = (\sqrt{2})^2$

or, $1+\frac{r}{100} = \sqrt{2}$ or, r = 100($\sqrt{2}$ – 1)

18.(c) Let, the savings of A and B be 4x, 5x and the share in cost of gift are 3y, 4y respectively
According to the question,

For A, $4x - 3y = \frac{2}{3}\times 4x$

$\Rightarrow x = \frac{9y}{4}$

For B, 5x – 4y = 145

$\Rightarrow 5\times\frac{9y}{4} - 4y = 145 \Rightarrow y = 20$

∴ Cost of gift = 3y + 4y = 7y
= 7 × 20 = 140

19.(a) Let, the weights of the pieces of diamond are x, 2x, 3x, 4x
∴ Total weight = 10x
∴ Total cost = $(10x)^2 = 100x^2$ ……(i)
∴ Cost of each piece = x^2, $4x^2$, $9x^2$, $16x^2$
∴ Total cost of the pieces = $30x^2$
∴ Total loss = $100x^2 - 30x^2 = 70x^2$
∴ $70x^2 = 70{,}000$
⇒ $x^2 = 1{,}000$
∴ Total cost of original diamond = $100x^2$
= 100 × 1000 = ₹ 100000

20.(c) Let, the number of undergraduate employees and graduate exployees be 13x and 23x respectively.
According to question, 30% of 13x = 351

$\Rightarrow \frac{30\times 13x}{100} = 351 \Rightarrow x = \frac{351\times 100}{390} = 90$

∴ Total number of undergraduate employees
= 13x = 13 × 90 = 1170
∴ Total number of graduate employees
= 23x = 23 × 90 = 2070

21.(a)

In vessel A	In Vessel B
$\text{Milk} = \frac{4}{7}$	$\text{Milk} = \frac{2}{5}$
$\text{Water} = \frac{3}{7}$	$\text{Water} = \frac{3}{5}$

Here both the vessels be mixed to obtain a new mix in vessel C containing half milk and half water.

$\frac{4}{7}$ $\frac{2}{5}$

$\frac{1}{2}$

$\frac{1}{10}$ $\frac{1}{14}$

14 : 10

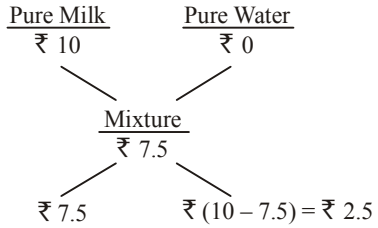
7 : 5

22.(d) Here, Son : Father : Mother
1 : 5
5 : 4
∴ Ratio of Son, father and Mother is 1 : 5 : 4
∴ Ratio of Son : Mother
1 : 4
and after 2 years the ratio becomes 3 : 10

$\therefore \frac{x+2}{4x+2} = \frac{3}{10}$

⇒ 10x + 20 = 12x + 6 ⇒ 2x = 14
∴ x = 7 ∴ Present age of father
5x = 5 × 7 = 35 yrs

23.(a) C.P = $9\times\frac{100}{120} = \frac{15}{2} = ₹\ 7.5$



Milk : Water = 7.5 : 2.5 = 3 : 1

24.(c) $P\times\frac{100}{112} = Q\times\frac{100}{96}$

$\Rightarrow \frac{Q}{P} = \frac{96}{112}$

∴ Q : P = 6 : 7

25.(b) S.P = ₹ 100 C.P = ₹ 80
Now, S.P = ₹ 90

Now, C.P $= 80 \times \frac{90}{100} =$ ₹ 72 অ্যাচিভার্স

$\therefore$ Profit percentage (%)

$= \frac{18}{72} \times 100\% = 25\%$

26.(c) Gain (%) in weight $= \frac{80}{920} \times 100\% = \frac{200}{23}\%$
Actual profit (%)

$= \left[15 + \frac{200}{23} + \frac{15 \times 200}{23 \times 100}\right]\%$

$= \left[\frac{345 + 200 + 30}{23}\right] = \frac{575}{23}\% = 25\%$

27.(b) $600 \times \frac{20}{100} = 120$

2 kg $\rightarrow$ 120
1 kg $\rightarrow$ 60

$\therefore$ Original price $60 \times \frac{100}{120} =$ ₹ 50

28.(a) $R = \frac{100(A_2 - A_1)}{A_1 t_2 - A_2 t_1} = \frac{100(650 - 600)}{6 \times 600 - 4 \times 650}$

$= \frac{100 \times 50}{1000} = 5\%$

29.(d) Sum of 6% $= \frac{100A - Pr_2 t}{(r_1 - r_2)t}$ অ্যাচিভার্স

$= \frac{100 \times 1600 - 7000 \times 4 \times 5}{(6-4) \times 5} =$ ₹ 2000

30.(b) $R = \left(\frac{I_2}{I_1} \times \frac{P_1}{P_2} \times r_1\right)\%$

$= \frac{2500}{4000} \times \frac{2000}{500} \times 4 = 10\%$

31.(d) 3% $\rightarrow$ ₹ 40 or, 1% $\rightarrow \frac{40}{3}$

Interest (S.I.) = A – P $= 80 - \frac{40}{3} = \frac{200}{3}$

$\therefore T = \frac{S.I \times 100}{P \times R} = \frac{200 \times 100 \times 3}{3 \times 40 \times 5} = 100$ years

32.(b) $CI - SI = \frac{50 \times 5}{200} =$ ₹ 1.25 অ্যাচিভার্স

33.(c) $CI = \frac{204}{200} \times SI$ $\therefore \frac{CI}{SI} = \frac{204}{200} = \frac{51}{50}$

34.(d) Amount $= 4800 \times \left(\frac{6000}{4800}\right)^{\frac{12}{4}} =$ ₹ 9375

35.(b) $1500\left(1 + \frac{25}{100}\right)^n > 2 \times 1500$ অ্যাচিভার্স

$\Rightarrow \left(\frac{5}{4}\right)^n > 2 \Rightarrow n = 4$

36.(c) $\text{Sum} = 2160\left[\frac{100}{120} + \left(\frac{100}{120}\right)^2 + \left(\frac{100}{120}\right)^3\right]$

$= 2160 \times \left[\frac{5}{6} + \frac{25}{36} + \frac{125}{216}\right] =$ ₹ 4550

37.(a) 4M = 8 W $\therefore$ 2M = 4W
4 M = 10 C
2M + 4W + 10C
= 2M + 2M + 4M = 8M
$M_1D_1 = M_2D_2$
$\Rightarrow 4 \times 2 = 8 \times D_2$ অ্যাচিভার্স
$\Rightarrow D_2 = 1$ day

38.(a) 1M = 2W
(8+2) M do work in day $= \frac{1}{6}$
1M does the work in day $= \frac{1}{60}$
8M and 4W did work in 2 days $= 2 \times \frac{1}{6} = \frac{1}{3}$
After 2 days 4M and 8W = 4M + 4M
= 8M do the work.
Remaining work $= 1 - \frac{1}{3} = \frac{2}{3}$

Time $= 60 \times \frac{2}{3} \times \frac{1}{8} = 5$ days

39.(a) Filling capacity $\Rightarrow$ x m^3 /min
Emptying capacity $\Rightarrow$ (x + 10)m^3/ min

ATQ, $\frac{2400}{x} - \frac{2400}{x+10} = 8$
$\Rightarrow x^2 + 10x - 3000 = 0$
$\Rightarrow x = 50$ m^3/min

40.(a) 72M = 144W = 180C
2M = 4W = 5C
4M + 12W + 20C = 18M
6M $\rightarrow$ 12 days অ্যাচিভার্স
18 M $\rightarrow$ 4 days
Remaining work $= \frac{1}{2}$

$\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$

$\frac{1}{2} \times 6 \times 12 = 1 \times M_2$

M_2 = 36 = No of man

41.(d) 2 × 6M = 18W = 24C

2M = 3W = 4C

3W + 4C =4M

Part of work done in 1 day by 4 M $= \frac{1}{3}$

Remaining $\frac{2}{3}$ work will be finished by 8M in 1 day.

Hence, 8M would be required

42.(b) Let , the tank fill in x min

$\frac{x}{36} + \frac{x}{45} - \frac{x-7}{30} = 1 \quad \therefore \ x = 46$

43.(a) Time taken by Car A = 2h

Car B met Car A at (9am + 2hrs) = 11am

Car B travelled for (11 am – 9.30 am)

$= \frac{3}{2}$ hrs

Speed of Car B = $90 \div \left(\frac{3}{2}\right)$ km/h = 60 km/hrs.

44.(a) Working hrs/day = 24 – 8 = 16h [24 hrs = 1 day]

Total working hrs in 20 days

= 16 × 20 = 320h

Now if distance and speed are doubled

Then = 24 – 16 = 8h

Reqd time $= \frac{320}{8} = 40$ days

45.(b) A.T.Q. $\frac{x}{50} = \frac{x+175}{75}$

x = 350

D = x + (x+175) = 875

46.(a) length of platform = 21 × 19 – 216 = 183

216 + 183 + 21n = 21 × 26

$\therefore$ n = 7

47.(c) $S = \frac{320}{16} = 20 \text{ m/s} = 20 \times \frac{18}{5} \text{km/h} = 72$

Total period of stoppage

= 5 × 18 = 90min $= \frac{3}{2}$h

Total time $= \left(\frac{576}{72} + \frac{3}{2}\right)$h $= 9\frac{1}{2}$h

48.(d) CP of pure spirit = ₹1

SP of mixture = ₹1

CP of mixture = $\frac{100}{100 + \frac{50}{3}} \times 1 = \frac{6}{7}$

Spirit		Water
1		0
	$\frac{6}{7}$	
$\frac{6}{7}$		$\frac{1}{7}$
6	:	1

49.(d) CP of mixture $= \frac{100}{125} \times 20 =$ ₹16

Chemical		Water
25		0
	16	
16	:	9

Water : Chemical = 9 : 16

50.(d) Quantity of milk in new mixture

$= 81\left(1 - \frac{1}{3}\right)^2 = 36$ lit

Quantity of water = 81 – 36 = 45 lit.

Milk : Water = 36 : 45 = 4 : 5

51.(d) $69\frac{3}{13}\% = \frac{9}{13}$

Mix-I		Mix-2
$\frac{8}{13}$		$\frac{5}{7}$
	$\frac{9}{13}$	
$\frac{2}{91}$		$\frac{1}{13}$
2	:	7

52.(d) L.C.M of (21, 35, 63) is–

$3 \overline{)21,\ 35,\ 63}$

$7 \overline{)7,\ 35,\ 21}$

$1,\ 5,\ 3$

$\therefore$ L.C.M = 3 × 7 × 5 × 3 = 315

$\therefore$ The numbers less than 10,000 which are divisible by 315 are given by $\left(\frac{10,000}{315}\right)$ i.e. the integral part when 10,000 is divided by 315 is 31

$\therefore$ The required number of numbers = 31

53.(a) Let, p(x) = $8(x^5 - x^3 + x)$

= 4 × 2 × x $(x^4 - x^2 + 1)$

and q(x) = 28 $(x^6 + 1)$

= 7 × 4 $[(x^2)^3 + (1)^3]$

$= 4 \times 7 \times (x^2 + 1)(x^4 - x^2 + 1)$
$[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$
$\because$ H.C.F of p(x) and q(x)
$= 4(x^4 - x^2 + 1)$

54.(d) Since $(x + 1)$ is the H.C.F of $Ax^2 + Bx + C$ and $(Bx^2 + Ax + C)$
$\therefore A(-1)^2 + B(-1) + C = 0$ [Put $x+1 = 0$]
$\Rightarrow A - B + C = 0$
$\therefore C = B - A$ ..... (i)
and $B(-1)^2 + A(-1) + C = 0$
$\Rightarrow B - A + C = 0 \Rightarrow C = A - B$ .... (ii)
Adding (i) and (ii)
$2C = 0 \Rightarrow C = 0$

55.(b) Let, $f(x) + g(x) = 5x^2 - x - 4$ …… (i)
and $f(x) - g(x) = x^2 + 9x - 10$ …… (ii)
On solving equation (i) and equation (ii) we get $f(x) = 3x^2 + 4x - 7$
$= (x - 1)(3x + 7)$
and $g(x) = 2x^2 - 5x + 3 = (x - 1)(2x - 3)$
Hence, the required H.C.F is $(x - 1)$

56.(c) Given H.C.F = $(x + y)$
L.C.M
$= 3x^5 + 5x^4y + 2x^3y^2 - 3x^2y^3 - 5xy^4 - 2y^5$
$= 3x^5 - 3x^2y^3 + 5x^4y - 5xy^4 + 2x^3y^2 - 2y^5$
$= 3x^2(x^3 - y^3) + 5xy(x^3 - y^3) + 2y^2(x^3 - y^3)$
$= (x^3 - y^3)(3x^2 + 5xy + 2y^2)$
and first polynomial
$= (x^2 - y^2) = (x + y)(x - y)$
We know that
First polynomial × Second polymial = H.C.F × L.C.M
$\therefore$ Second Polynomial

$$= \frac{(x+y)(x^3-y^3)(3x^2+5xy+2y^2)}{(x+y)(x-y)}$$

$$= \frac{(x-y)(x^2+xy+y^2)(3x^2+5xy+2y^2)}{(x-y)}$$

$= (x^2 + y^2 + xy)(3x^2 + 5xy + 2y^2)$
$= 3x^4 + 5x^3y + 2x^2y^2 + 3x^2y^2 + 5xy^3 + 2y^4 + 3x^3y + 5x^2y^2 + 2xy^3$
$= 3x^4 + 8x^3y + 10x^2y^2 + 7xy^3 + 2y^4$

57.(c) Here,
A's investment = B's investment + 700
B's investment = C's investment + 500
Let say, C's investment is x
B's investment = (x + 500)
Then, A's investment
= (x + 500 + 700) = (x + 1200)
According to the sum,
$x + (x + 500) + (x + 1200) = 4700$
$\Rightarrow 3x = 3000 \Rightarrow x = 1000$
Investments of A, B and C are respectively 2200, 1500 and 1000
Profit ratio of A, B and C
= 2200 : 1500 : 1000 = 22 : 15 : 10
C's share of profit = $\left(\frac{10}{47} \times 423\right) = 90$

58.(c) Ratio of profit = (2500×11) : (4500×11) = 5 : 9
then, (9–5) = 4 units = 4800
$(9+5) \rightarrow 14$ units $= \left(4800 \times \frac{14}{4}\right) = 16800$
$\therefore$ Total annual profit is 16800

59.(b)

A's share	:	B's share	:	C's share
(42×4)+(30×6)	:	(30×4)+(24×6)	:	(28×4)+(20×6)
348	:	264	:	232
87	:	66	:	58

$\therefore$ Then share of C is $\left(\frac{58}{211} \times 46420\right) = 12760$

60.(d) A's share : (100×6)+(150×6)
B's share : (200×6)+(600×6)
C's share : (400×6)+(300×6)

A's share	:	B's share	:	C's share
5	:	16	:	14

$\therefore$ The ratio in which the profit is to be divided is 5 : 16 : 14

61.(b) Suppose A puts in 2x and B puts in x
Thus, A : B profit ratio is –

$$(2x \times 3) + \left(\frac{4}{3}x \times 4\right) + \left(\frac{14x}{9} \times 5\right) : (x \times 7) + \left(\frac{3}{4}x \times 5\right)$$

$$= \left(6x + \frac{16x}{3} + \frac{70x}{9}\right) : \left(7x + \frac{15x}{4}\right)$$

$$= \frac{172x}{9} : \frac{43x}{4} = 16 : 9$$

Now, according to the sum,
A received 320 as profit
16 units = 320
9 units = 180
B received 180 as profit

62.(c) $\frac{2}{3}\pi \times 6^3 = \frac{1}{3}\pi \times r^2 \times 75$

$$\Rightarrow r^2 = \frac{2 \times 6 \times 6 \times 6}{75} = \frac{4 \times 36}{25}$$

$$\therefore r = \sqrt{\frac{4 \times 36}{25}} = \frac{2 \times 6}{5} = \frac{12}{5} = 2.4 \text{ cm}$$

63.(b) No. of men $= \frac{24 \times 15 \times \frac{1}{100}}{0.1} = \frac{24 \times 15}{10} = 36$

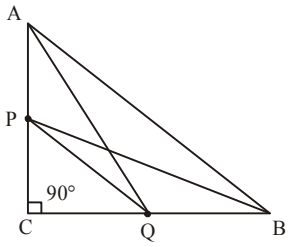
64.(c) Field raised $= \frac{5 \times 4.5 \times 2.1}{(13.5 \times 2.5) - (5 \times 4.5)}$

$= \frac{5\times 4.5\times 2.1}{11.25} = 4.2$ m

65.(b) Ball is dropped from the height of 36m when ball rises at the third bounce.

Required height $= \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times 36 = \frac{32}{3} = 10\frac{2}{3}$ m

66. (b)



$AQ^2 = AC^2 + QC^2$

$BP^2 = BC^2 + PC^2$

So, $AQ^2+BP^2 = (AC^2 + BC^2) + (QC^2+PC^2)$

$\Rightarrow AQ^2 + BP^2 = (AB)^2 + (PQ)^2$

$= (AB)^2 + (\frac{1}{2}AB)^2$

$\Rightarrow AQ^2 + BP^2 = AB^2 + \frac{AB^2}{4} = \frac{5AB^2}{4}$

$\Rightarrow 4(AQ^2 + BP^2) = 5AB^2$

67. (c) $BD\perp AC$ and $AB\perp BC$

So, AC $= \sqrt{AB^2+BC^2} = \sqrt{a^2+b^2}$

Again, area of ΔABC

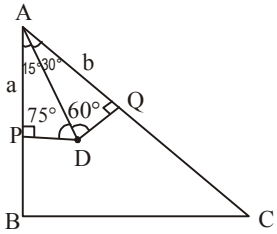
$\Rightarrow$ ½ × AB × BC = ½ × AC × BD

$\Rightarrow$ AB × BC = AC × BD

$\Rightarrow a \times b = \sqrt{a^2+b^2}\times p$

$\Rightarrow a^2b^2 = p^2(a^2 + b^2) \Rightarrow p^2 = \frac{a^2b^2}{a^2+b^2}$

68. (c)



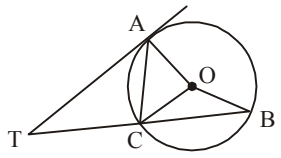
From, ΔAQD, $\frac{AQ}{AD} = \sin 60°$

$\Rightarrow \frac{b}{AD} = \frac{\sqrt{3}}{2} \Rightarrow AD = \frac{2b}{\sqrt{3}}$

From ΔAPD,

$\sin 75° = \frac{AP}{AD} \Rightarrow \sin 75° = \frac{a\sqrt{3}}{2b}$

69. (d)



$\angle ACB = 40° + 44° = 84°$

$\angle ACO = 90° - 44° = 46° = \angle OAC$

Again, $\angle OCB = \angle ACB - \angle ACO$

$= 84° - 46° = 38° = \angle OBC$

$\therefore \angle BOC = 180° - (38° + 38°) = 104°$

70. (d) PDC = ½ ∠ADC and ∠PCD = ½ ∠BCD

So, ∠PDC+∠PCD= ½ (∠ADC+∠BCD)

= ½ × 180° = 90° [as, sum of two consecutive angles of a parallelogram is 180°]

In ΔPCD,

$\Rightarrow x + \angle PDC + \angle PCD = 180°$

$\Rightarrow x = 180° - (\angle PDC + \angle PCD)$

$\Rightarrow x = 180° - 90° = 90°$

71. (c) $\angle AOB = 2 \times \angle ACB = 2 \times 65° = 130°$

Now, OA = OB (r)

AP = BP (tangents from same point P)

OP is common

$\therefore \Delta AOP \cong \Delta BOP$ (s–s–s)

So, $\angle AOP = \angle BOP = 65°$

Hence, $\angle APO = 90° - 65° = 25°$

72. (b) Let, G be the mid-point of FC.

In ΔBCF, D is the mid-point of BC and G is the mid-point of FC. So, DG || EF

In ΔADG, E is the mid-point of AD and EF||DG.

So, F is the mid-point of AG

$\therefore$ AF = FG = GC

Hence, AF $= \frac{1}{3}AC$

73. (b) In ΔAOB, OA + OB > AB

In ΔBOC, OB + OC > BC

In ΔAOC, OA + OC > AC

$\therefore 2(OA + OB + OC) > (AB + BC + CA)$

$\therefore (OA + OB + OC) > ½ (AB+BC+CA)$

74. (a) $AB^2 = AC^2 - BC^2 = 25 - BC^2$ .... (i)

And, $AB^2 = AD^2 - BD^2$

$= \left(\frac{3\sqrt{5}}{2}\right)^2 - \left(\frac{BC}{2}\right)^2 = \frac{45}{4} - \frac{BC^2}{4}$ ...(ii)

From (i) & (ii),

$25 - BC^2 = \frac{45}{4} - \frac{BC^2}{4}$

$\frac{3BC^2}{4} = \frac{55}{4} \Rightarrow BC^2 = \frac{55}{3}$

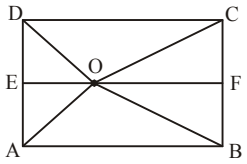
So, $AB^2 = 25 - \frac{55}{3} = \frac{20}{3}$

Again, $CE^2 = BC^2 + BE^2$

$= BC^2 + \left(\frac{AB}{2}\right)^2 = \frac{55}{3} + \frac{5}{3} = \frac{60}{3}$ অ্যাচিভার্স

$\Rightarrow CE^2 = 20 \quad \Rightarrow CE = 2\sqrt{5}$

75. (d)



Draw EF || AB
In ΔAOE and ΔCOF
$OA^2 = OE^2 + AE^2$ and $OC^2 = OF^2 + CF^2$
$\therefore OA^2 + OC^2$
$= OE^2 + AE^2 + OF^2 + CF^2$ ... (i)
Again, in ΔDOE and ΔBOF
$OD^2 = OE^2 + DE^2$ and $OB^2 = OF^2 + BF^2$
$\therefore OD^2 + OB^2$
$= OE^2 + DE^2 + OF^2 + BF^2$ ... (ii)
As, AE = BF and DE = CF
From (i) & (ii), we can say
$OA^2 + OC^2 = OD^2 + OB^2$

76. (d) $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$

$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} \quad \left[\because \text{cosec}\,\theta = \frac{1}{\sin\theta}; \sec\theta = \frac{1}{\cos\theta}\right]$

$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20°.\cos 20°}$

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$= \frac{2 \times 2\left(\frac{\sqrt{3}}{2}.\cos 20° - \frac{1}{2}.\sin 20°\right)}{2.\sin 20°.\cos 20°}$

$= \frac{4(\sin 60°.\cos 20° - \cos 60°.\sin 20°)}{2.\sin 20°.\cos 20°}$

$= \frac{4\sin(60° - 20°)}{\sin 40°} = \frac{4\sin 40}{\sin 40°} = 4$

77. (b) $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$= 1 + \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} - 1$

$= \left(1 + \cos\frac{2\pi}{7}\right) + \left(\cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right) - 1$

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$= 2\cos^2\frac{\pi}{7} + \left(\cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right) - 1$

$[\because 2\cos^2\theta = 1 + \cos 2\theta]$

$= 2\cos^2\frac{\pi}{7} + 2.\cos\frac{5\pi}{7}.\cos\frac{\pi}{7} - 1$ অ্যাচিভার্স

$\left[\because \cos C + \cos D = 2\cos\frac{(C+D)}{2}.\cos\frac{(C-D)}{2}\right]$

$= 2\cos\frac{\pi}{7}\left(\cos\frac{\pi}{7} + \cos\frac{5\pi}{7}\right) - 1$

$= 4\left(\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{3\pi}{7}\right) - 1$

Now,

$\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{3\pi}{7}$

$= \frac{1}{\sin\frac{\pi}{7}}.\sin\frac{\pi}{7}.\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{3\pi}{7}$

$= \frac{1}{2}.\frac{1}{\sin\frac{\pi}{7}}.2\sin\frac{\pi}{7}.\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{3\pi}{7}$

$= \frac{1}{2\sin\frac{\pi}{7}}.\sin\frac{2\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{3\pi}{7}$

$[\because \sin 2\theta = 2\sin\theta.\cos\theta]$

$= \frac{1}{2.2\sin\frac{\pi}{7}}.2\sin\frac{2\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{3\pi}{7}$

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$= \frac{1}{4\sin\frac{\pi}{7}}.\sin\frac{4\pi}{7}.\cos\frac{3\pi}{7}$

$= \frac{1}{4.\sin\frac{\pi}{7}}.\sin\frac{3\pi}{7}.\cos\frac{3\pi}{7}$

$\left[\because \sin\frac{4\pi}{7} = \sin\left(\pi - \frac{3\pi}{7}\right) = \sin\frac{3\pi}{7}\right]$

$= \frac{1}{8.\sin\frac{\pi}{7}}.2\sin\frac{3\pi}{7}.\cos\frac{3\pi}{7}$

$= \frac{1}{8.\sin\frac{\pi}{7}}.\sin\frac{6\pi}{7} = \frac{1}{8.\sin\frac{\pi}{7}}.\sin\left(\pi - \frac{\pi}{7}\right)$

$[\because \sin(180° - \theta) = \sin\theta]$

$= \frac{1}{8}.\frac{1}{\sin\frac{\pi}{7}}.\sin\frac{\pi}{7} = \frac{1}{8}$

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Hence,

$4\left(\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{3\pi}{7}\right) - 1 = \left(4.\frac{1}{8}\right) - 1 = -\frac{1}{2}$

78. (b) Let, $\theta = 60°$

Now, $\frac{1 - \tan(90° - 60°) + \sec(90° - 60°)}{\tan(90° - 60°) + \sec(90° - 60°) + 1}$

$$= \frac{1-\tan 30^\circ + \sec 30^\circ}{\tan 30^\circ + \sec 30^\circ + 1}$$

$$= \frac{1-\frac{1}{\sqrt{3}}+\frac{2}{\sqrt{3}}}{\frac{1}{\sqrt{3}}+\frac{2}{\sqrt{3}}+1} = \frac{1+\frac{1}{\sqrt{3}}}{\sqrt{3}+1}$$ অ্যাচিভার্স

$$= \frac{(\sqrt{3}+1)}{\sqrt{3}(\sqrt{3}+1)} = \frac{1}{\sqrt{3}}$$

$$\therefore \tan\frac{\theta}{2} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$\therefore$ given expression $= \tan\frac{\theta}{2}$

79. (d) let y = 0

$$\frac{(\sin 4x + \sin 4y).\tan(2x-2y)}{\sin 4x - \sin 4y}$$

$$= \frac{\sin 4x.\tan 2x}{\sin 4x} = \tan 2x$$

$\therefore$ put y = 0 in option; we get from tan(2x+2y) = tan 2x

Given expression = tan (2x +2y)

80. (a) $\frac{\cos 3\theta + 2\cos 5\theta + \cos 7\theta}{\cos\theta + 2\cos 3\theta + \cos 5\theta} + \sin 2\theta.\tan 3\theta$ অ্যাচিভার্স

$$= \frac{(\cos 3\theta + \cos 7\theta) + 2\cos 5\theta}{(\cos\theta + \cos 5\theta) + 2\cos 3\theta} + \sin 2\theta.\tan 3\theta$$

$$= \frac{2\cos 5\theta.\cos 2\theta + 2\cos 5\theta}{2\cos 3\theta.\cos 2\theta + 2\cos 3\theta} + \sin 2\theta.\tan 3\theta$$

$$= \frac{2\cos 5\theta(\cos 2\theta + 1)}{2\cos 3\theta(\cos 2\theta + 1)} + \frac{\sin 2\theta.\sin 3\theta}{\cos 3\theta}$$

$$= \frac{\cos(3\theta + 2\theta) + \sin 2\theta.\sin 3\theta}{\cos 3\theta}$$

$$= \frac{(\cos 3\theta.\cos 2\theta - \sin 3\theta.\sin 2\theta + \sin 3\theta.\sin 2\theta)}{\cos 3\theta}$$

= cos2θ

81. (b) $\frac{\cos 2\alpha}{1} = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$

$$\Rightarrow \frac{\cos 2\alpha + 1}{\cos 2\alpha - 1} = \frac{3\cos 2\beta - 1 + 3 - \cos 2\beta}{3\cos 2\beta - 1 - 3 + \cos 2\beta} = \frac{2\cos 2\beta + 2}{4\cos 2\beta - 4}$$

$$\Rightarrow \frac{2\cos^2\alpha}{-2\sin^2\alpha} = \frac{2(\cos 2\beta + 1)}{4(\cos 2\beta - 1)} \quad \because\ 2\cos^2\alpha = (\cos 2\alpha + 1)$$

$$\Rightarrow \frac{\cos^2\alpha}{-\sin^2\alpha} = \frac{2}{4}\left(\frac{2\cos^2\beta}{-2\sin^2\beta}\right)$$

$$\Rightarrow \frac{\cos^2\alpha}{\sin^2\alpha} = \frac{1}{2}\left(\frac{\cos^2\beta}{\sin^2\beta}\right)$$ অ্যাচিভার্স

$$\Rightarrow \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{2\sin^2\beta}{\cos^2\beta}$$ অ্যাচিভার্স

$\tan^2\alpha = 2\tan^2\beta \Rightarrow \tan\alpha = \sqrt{2}\tan\beta$

82. (a) Let, put θ = 0°, then we can write

$$x\cos 0^\circ = y\cos\left(\frac{2\pi}{3}\right) = z\cos\frac{4\pi}{3}$$

x= y cos120° = z cos240° [$\because$cos 0° = 1]

x = y cos (180°–60°) = z cos (270°–30°)

x = y(–cos 60°) = z (–sin30°)

$$x = -\frac{y}{2} = -\frac{z}{2}$$

[$\because$cos(π–θ) = –cosθ, cos (270°– θ) = –sin θ]

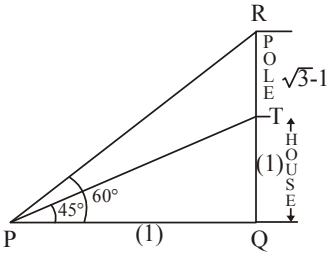
$$x = -\frac{y}{2} = -\frac{z}{2} = K \text{ (Let)}$$

Now, x = K; y = –2k; z = –2k

xy + yz + zx

= $-2k^2 + 4k^2 - 2k^2 = (4k^2 - 4k^2) = 0$

83. (c)



TQ = Length of house = 25m

RT = length of Pole

In ΔPTQ , ∠TPQ = 45° অ্যাচিভার্স

$$\therefore \tan 45^\circ = 1 = \frac{TQ}{PQ}$$

Now, TQ = 1; PQ = 1

Then, ΔPQR, ∠RPQ = 60°;

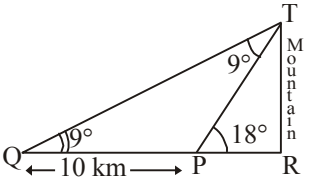
$$\tan 60^\circ = \frac{RQ}{PQ} = \frac{\sqrt{3}}{1}$$

$RQ = \sqrt{3}$, PQ = 1 then $RT = (\sqrt{3}-1)$

According to picture; if TQ = 1 ≡ 25

Then $RT = (\sqrt{3}-1) = 25(\sqrt{3}-1)$m

84. (a)



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In ΔTQP; ∠PTQ = ∠PQT = 9°

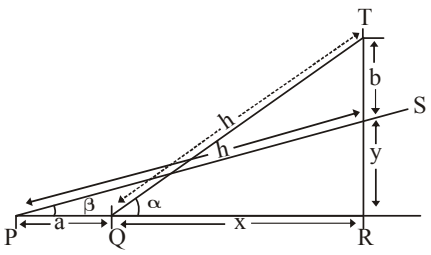
$\therefore$ PQ = PT = 10km

In ΔTPR, $\sin 18° = \frac{TR}{PT} = \frac{TR}{10}$

$[\because \sin 18° = \frac{\sqrt{5}-1}{4}]$

$\frac{\sqrt{5}-1}{4} = \frac{TR}{10} \Rightarrow TR = \frac{5}{2}(\sqrt{5}-1) = 2.5(\sqrt{5}-1)$

85. (a) অ্যাচিভার্স



Let, length of ladder is h
$\therefore$ TQ = PS = ladder = h

In ΔTRQ : $\sin\alpha = \frac{b+y}{h}$ and

ΔPSR : $\sin\beta = \frac{y}{h}$

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$\therefore \sin\alpha - \sin\beta = \left(\frac{b}{h} + \frac{y}{h} - \frac{y}{h}\right) = \frac{b}{h} \rightarrow (i)$

Now, ΔTRQ : $\cos\alpha = \frac{x}{h}$ and ΔPSR : $\cos\beta = \frac{a+x}{h}$

$\therefore \cos\beta - \cos\alpha = \left(\frac{a}{h} + \frac{x}{h} - \frac{x}{h}\right) = \frac{a}{h} \rightarrow (ii)$

Now, (ii) ÷ (i), we get $\rightarrow \frac{a}{b} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$

$[\because \beta > \alpha]$

$\Rightarrow \frac{a}{b} = \frac{2\sin\frac{\alpha+\beta}{2}.\sin\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2}.\sin\frac{\alpha-\beta}{2}}$

$\Rightarrow \frac{a}{b} = \tan\left(\frac{\alpha+\beta}{2}\right)$

86. (c) $x^2 + y^2 + z^2 = 2(x + z - 1)$
$x^2 + y^2 + z^2 = 2x + 2z - 2$
$x^2 + y^2 + z^2 = 2x + 2z - 1 - 1$
$(x^2 - 2x + 1) + (z^2 - 2z + 1) + y^2 = 0$
$(x - 1)^2 + (z - 1)^2 + y^2 = 0$
If sum of the square is zero; then the factors are individually zero.

$(x-1)^2 = 0 \Rightarrow \therefore x = 1$ | $(z-1)^2 = 0 \Rightarrow \therefore z = 1$ | $y^2 = 0 \Rightarrow \therefore y = 0$

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Putting the values of x, y and z in e.q $x^3 + y^3 + z^3$; we get the ans is 2

87. (d) x, y and z are natural numbers.

So, x =1, $y^{z^x} = 125 = (5)^{3^1}$; $z^{y^x} = 3^{5^1}$

x = 1, y = 5 and z = 3

Now 9x + 10y – 18z

= (9×1) + (10×5) – (18 ×3) = 5

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88. (d) $N = 3^{14} + 3^{13} - 12 \Rightarrow N = 3^{13}(3+1) - 12$
$\Rightarrow N = 3.4.3^{12} - 12 \Rightarrow N = 12(3^{12}-1)$
$N = 12[(3^6)^2 - (1)^2]$
$\Rightarrow N = 12\ [(3^6+1)\ (3^6-1)]$
$\Rightarrow N = 12\ [(729+1)\ (729-1)]$
$N = 12 \times 730 \times 728$
$\therefore N = 12 \times 73 \times 10 \times 728$
$\therefore$ The largest prime factor is 73

89. (b) Let, a = b = 1
Putting the value of a and b

Now, $\left(\frac{1}{a^2}\right) + \left(\frac{1}{b^2}\right) + \left(\frac{a}{b}\right) + \left(\frac{b}{a}\right) = 4$

$\therefore$ (x – a) (x–b) = 0 [Q a = b = 1]
$\Rightarrow (x-1)^2 = 0$
$\Rightarrow x^2 - 2x + 1 = 0$
by compairing with $Px^2 - QX + R = 0$
$\therefore$ P = 1 ; Q = 2; R = 1
Put the value of P, Q, and R in given option; then from the options we have,
Option (a) 6
Option (b) 4
Option (c) 6
Option (d) 8
The required answer is b

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90. (c) Let, a = b = c
$\therefore$ The given expression becomes

$\frac{3(1-a^2)^2}{a^2}$.....(i)

a + b + c = abc $\Rightarrow 3a = a^3 \Rightarrow a^2 = 3$
Put the value of a^2 in eq (i), we get

$\frac{3(1-a^2)^2}{a^2} = \frac{3(1-3)^2}{3} = 4$

91. (b) Expenditure on clothing = ₹825
$\because$ 36° ≡ ₹825

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$\therefore$ 360° ≡ $\frac{825}{36} \times 360$ = ₹8250

92. (a) Corresponding angle for savings = 54°
$\because$ 360° ≡ 100%

$\therefore\ 54° \equiv \frac{100}{360} \times 54 = 15\%$

93. (c) Required ratio = 108 : 72 = 3 : 2

94. (a) Total expenditure = ₹8250

$\therefore$ Expenditure on clothes and rent

$= \frac{8250 \times (36° + 90°)}{360°} = \frac{8250 \times 126}{360}$

= ₹ 2887.5

$\therefore$ Average expenditure $= \frac{2887.5}{2}$

= ₹1443.75

95. (d) Required ratio

$= \left(\frac{108 + 36 + 72}{3}\right) : \left(\frac{54 + 90}{2}\right)$

$= \frac{216}{3} : \frac{144}{2} = 72 : 72 = 1 : 1$

96. (a) Average production of type P vehicles

$= \frac{100 + 125 + 200 + 225 + 275 + 275}{6}$

$= \frac{1200}{6} = 200$ thousands

Required years $\Rightarrow$ 2012, 2013 and 2014

97. (b) Required percentage decrease

$= \left(\frac{150 - 125}{150}\right) \times 100 = \frac{25}{150} \times 100 = \frac{50}{3}$

= 16.7%

98. (c) Total production of type P vehicles in 2009 and 2011 = 100 + 200 = 300 thousands

Total production of type Q vehicles in 2010 and 2014 = 150 + 225 = 375 thousands.

$\therefore$ Required percent $= \frac{300}{375} \times 100\% = 80\%$

99. (a) Total production of type P vehicles = 1200 thousands.

Total production of type Q vehicles

= 175 + 150 + 125 + 175 + 175 + 225 = 1025

$\therefore$ Required ratio = 1200 : 1025 = 48 : 41

100. (c) Required percentage

$= \frac{150}{275} \times 100\% = 54.5\%$

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